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NUMERICAL ABILITY / AFCAT

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Decimal Fraction

Fractions, decimal expansions and reciprocals

01 Operate with fractions

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

Denominators and the divisor c/d must be nonzero.

02 Recognise a terminating decimal

$$\frac{p}{q} \text{ terminates} \Leftrightarrow q = 2^a 5^b$$

Reduce p/q to lowest terms with q > 0; a,b are nonnegative integers.

03 Turn recurring decimals into fractions

$$0.\overline{abc} = \frac{\overline{abc}}{999}$$

$$0.a\overline{b} = \frac{10a + b - a}{90}$$

The barred numerator is the three-digit block, not a product. The second formula has one nonrepeating and one repeating digit.

Quick method. Cross-cancel before multiplying fractions. For a recurring decimal, shift the repeating tail into alignment and subtract the two equations.

EXAMPLE 1 **Convert 0.1 followed by recurring 6 to a fraction.**

Let x = 0.1666... . Then 10x = 1.666... and 100x = 16.666...; subtraction removes the recurring tail.

$$90x = 15 \Rightarrow x = \frac{1}{6}$$

EXAMPLE 2 **Does 21/140 terminate, and what is its decimal value ?**

Reduce to 3/20. The denominator contains only 2 and 5.

$$\frac{21}{140} = \frac{3}{20} = 0.15$$

Watch for. The terminating-decimal test applies to the reduced denominator. A percentage is a fraction out of 100, not an extra unit to multiply twice.

Time and Work

Time, work, efficiency and pipes

01 Think in rates

$$\text{rate} = \frac{1}{\text{time}}, \qquad \text{work} = \text{rate} \times \text{time}$$

For total work normalised to 1, rate = 1/time; otherwise use work/time. Add constant rates only while workers/pipes act together.

02 Combine two positive work rates

$$T_{AB} = \frac{ab}{a+b}$$

A and B separately finish the same work in a and b positive time units.

03 Relate efficiency to completion time

$$\text{efficiency ratio } m : n \Rightarrow \text{time ratio } n : m$$

For the same total work; an outlet or destroyer has a negative contribution.

Quick method. Choose the LCM of individual times as the total work, making hourly or daily rates integers. For changing teams, compute completed work stage by stage.

EXAMPLE 1 **A can finish a job in 12 days and B in 18 days. How long do they take together ?**

Take total work as 36 units. Their daily rates are 3 and 2 units, totalling 5.



AFTER ONE DAY A: 3 units + B: 2 units = 5 of 36 units complete

$$T = \frac{36}{5} = 7.2 \text{ days}$$

EXAMPLE 2 **A pipe fills a tank in 6 hours while a leak empties a full tank in 9 hours. How long to fill an empty tank with both acting ?**

Subtract the leak rate from the inlet rate, assuming both rates are constant.

$$\text{net rate} = \frac{1}{6} - \frac{1}{9} = \frac{1}{18} \Rightarrow T = 18 \text{ h}$$

Watch for. Add rates, not completion times. An outlet can make the net rate zero or negative, in which case the tank will not fill under those conditions.

Time and Work

Changing teams and alternate-day work

01 Account for completed work

$$W_{\text{done}} = \sum_i r_i t_i, \qquad W_{\text{left}} = 1 - W_{\text{done}}$$

Rates are fractions of the same whole job per time unit.

02 Finish the remaining fraction

$$T_{\text{final}} = W_{\text{left}} / r_{\text{final}}$$

Assume a positive net rate during the final stage.

03 Use complete alternate-day cycles

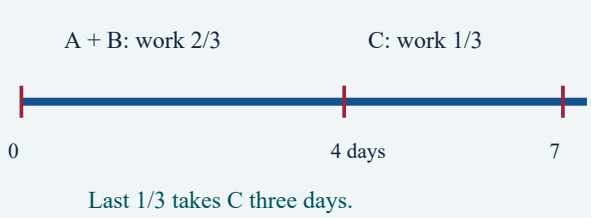
$$W_{\text{cycle}} = r_A + r_B$$

For one-day turns by A and B, a full two-day cycle completes this work fraction.

Quick method. Calculate completed work first, then the remaining fraction. For alternate turns, use whole cycles and inspect the final day separately.

EXAMPLE 1 **A finishes a job in 10 days and B in 15. They work together for four days, then C finishes the rest in three days. Find C's solo completion time.**

A and B complete 4(1/10 + 1/15) = 2/3. C does 1/3 in three days.



Calculate work already completed first.

Do not apply the original joint rate later.

C alone needs three times three days.

$$r_C = \frac{1}{9} \Rightarrow T_C = 9 \text{ days}$$

EXAMPLE 2 **A takes six days and B three days. They work alternate days, A first. Find the completion time.**

Each two-day cycle completes 1/2. After three days the work done is 2/3; B finishes 1/3 on day four.

$$T = 4 \text{ days}$$

Watch for. The worker who starts can change the finish time. Do not round a partial final day up unless the question explicitly asks for whole attendance days.

Time and Work

Workers, hours and output

01 Measure output in worker-hours

$$Q = kMDH$$

Output Q is proportional to workers M, days D and hours/day H at fixed productivity k.

02 Scale the number of days

$$\frac{D_2}{D_1} = \frac{Q_2}{Q_1}\frac{M_1}{M_2}\frac{H_1}{H_2}$$

Assume identical worker efficiency and comparable output units.

03 Allow for different efficiencies

$$Q = kEMDH$$

Include relative efficiency E when teams differ in productivity.

Quick method. Set up the output-per-worker-hour equation before multiplying. A longer wall or road means more work only when its other dimensions and construction remain comparable.

EXAMPLE 1 Twelve workers make 180 units in 15 days at six hours/day. How many days do ten workers need for 240 units at eight hours/day ?

Scale days by output, inverse worker count and inverse hours.

OUTPUT

workers x days x hours

x relative efficiency

Output rises with worker-hours.

Time falls as effective workforce rises.

Keep productivity and job units comparable.

$$D = 15 \cdot \frac{240}{180} \cdot \frac{12}{10} \cdot \frac{6}{8} = 18$$

EXAMPLE 2 Eight workers finish a job in 15 days. Ten new workers are each 20% more efficient. Find their completion time at the same daily hours.

Old effort is 8 times 15. New daily effective workforce is 10 times 1.2.

$$D = \frac{8 \cdot 15}{10 \cdot 1.2} = 10$$

Watch for. More output increases time; more workers or longer days reduce it. Check each ratio's direction independently.

Average

Means, weighted averages and corrections

01 Calculate ordinary and weighted means

$$\bar{x} = \frac{\sum x_i}{n}, \qquad \bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}$$

Positive total weight is required. Percentage weights may be used as fractions.

02 Combine groups of unequal sizes

$$\bar{x}_{\text{combined}} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Group sizes determine the weights; do not average unequal-sized group means equally.

03 Correct a wrongly recorded value

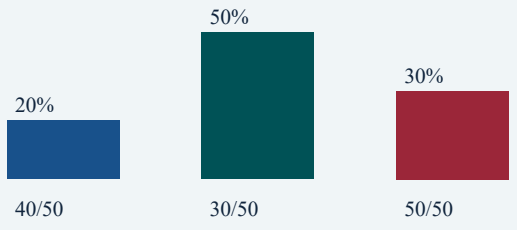
$$\bar{x}_{\text{corrected}} = \bar{x}_{\text{old}} + \frac{\text{correct} - \text{wrong}}{n}$$

Replace one erroneous entry in a data set with n observations.

Quick method. Recover the total as mean times count. When two weights swap, use the difference in weights times the difference in values to find the mean change.

EXAMPLE 1 **Three tasks score 40,30,50 out of 50 with weights 20%,50%,30%. Find the weighted score and the increase when the first two weights swap.**

Original score is 8 + 15 + 15 = 38. The swap adds (0.5 - 0.2)(40 - 30) = 3.



Original weights are 20%, 50%, 30%.

Score = 0.2(40) + 0.5(30) + 0.3(50).

The weighted result stays out of 50.

$$\bar{x} = 38, \qquad \bar{x}_{\text{new}} = 41$$

EXAMPLE 2 **The mean of 12 observations is 25. One entry was recorded as 18 instead of 42. Find the corrected mean.**

The total must increase by 24; spread this correction over 12 observations.

$$\bar{x} = 25 + 24/12 = 27$$

Watch for. A mean price per kg is total cost divided by total kg. A raw weighted score out of 50 is not automatically a percentage out of 100.

Percentage

Percentage bases, income and savings

01 Apply or reverse a percentage change

$$\text{new} = \text{old}(1 + p/100), \quad \text{original} = \frac{\text{final}}{1 + p/100}$$

Use a negative p for a decrease; the reversing factor must be nonzero.

02 Combine successive changes

$$\Delta\% = p + q + pq/100$$

Successive signed changes multiply their factors; they do not simply add.

03 Separate income, spending and savings

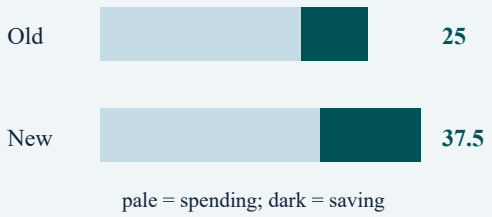
$$S_0 = (1 - e)I, \quad S_1 = [1 + a - e(1 + b)]I$$

Original expenditure is fraction e of income I; income/expenditure increase by fractions a,b.

Quick method. Choose income 100 units when only percentages are given. For elections, apply turnout, valid-vote fraction and the winner's share in separate stages.

EXAMPLE 1 A person spends 75% of income. Income rises by 20% and expenditure by 10%. Find the percentage rise in savings.

Take old income 100: expenditure 75 and savings 25. New savings are 120 - 82.5 = 37.5.



Use original income 100 as the base.

New income 120; new spending 82.5.

Savings grow from 25 to 37.5: up 50%.

$$\frac{37.5 - 25}{25} \cdot 100 = 50\%$$

EXAMPLE 2 In a two-candidate election, 75% vote, 20% of polled votes are invalid, and the winner gets 60% of valid votes. A margin of 360 votes represents how many registered voters ?

The margin is 20% of valid votes, which are 60% of registered voters. Thus 12% of the electorate equals 360.

$$N = \frac{360}{0.75 \cdot 0.8 \cdot 0.2} = 3000$$

Watch for. Savings growth is measured against old savings, not old income. A winner's margin is the difference between the two candidates' valid votes.

Profit & Loss

Profit, loss, marked price and discount

01 Use cost price as the profit base

$$\text{profit}\% = 100\frac{SP - CP}{CP}, \quad SP = CP\left(1 + \frac{p}{100}\right)$$

Profit/loss percentage is based on cost price.

02 Apply a marked-price discount

$$SP = MP\left(1 - \frac{d}{100}\right)$$

Discount percentage is based on marked price.

03 Combine successive discounts

$$d_{\text{equivalent}} = d_1 + d_2 - \frac{d_1 d_2}{100}$$

For two successive discounts on the reducing marked amount.

Quick method. Track CP, MP and SP separately. In dishonest-weight questions, compare the amount paid for with the amount actually supplied.

EXAMPLE 1 **An article costing 800 is marked 25% above cost and sold at a 10% discount. Find the profit percentage.**

The marked price is 1000, and selling price is 900. The gain is 100 on a cost of 800.

$$\text{profit}\% = \frac{100}{800} \cdot 100 = 12.5\%$$

EXAMPLE 2 **Two articles are each sold for 1200, one at 20% profit and the other at 20% loss. Find the overall result.**

Their costs are 1000 and 1500. Total selling price is 2400 against total cost 2500.

$$\text{loss}\% = \frac{100}{2500} \cdot 100 = 4\%$$

Watch for. Equal profit and loss percentages on two items sold at the same price do not cancel: the cost prices differ.

Ratio & Proportion

Ratio, proportion and variation

01 Represent a ratio by one multiplier

$$a : b = m : n \Rightarrow a = km, \quad b = kn$$

A common multiplier preserves the ratio.

02 Distinguish direct and inverse variation

$$y \propto x \Rightarrow y = kx, \quad y \propto 1/x \Rightarrow xy = k$$

Direct and inverse variation have different invariants.

03 Compare compound variations

$$y \propto \frac{x^a}{z^b} \Rightarrow \frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^a \left(\frac{z_1}{z_2}\right)^b$$

Hold the proportionality constant fixed and use positive quantities where needed.

Quick method. Equalise the shared term when joining two ratios. In a work/production table, decide which variables vary directly and which inversely before multiplying ratios.

EXAMPLE 1 **A:B = 2:3 and B:C = 4:5. Find A:B:C.**

Make B equal to 12 in both ratios: multiply the first by 4 and the second by 3.

$$A : B : C = 8 : 12 : 15$$

EXAMPLE 2 **y varies directly as x^2 and inversely as z. It is 12 at x = 2, z = 3. Find y at x = 3, z = 6.**

Use the ratio of the two situations to avoid solving separately for the constant.

$$y = 12\left(\frac{3}{2}\right)^2 \frac{3}{6} = \frac{27}{2}$$

Watch for. Equal changes in two quantities do not preserve their ratio. Ratios need consistent units.

Ratio & Proportion

Capital-time shares and changing investments

01 Weight capital by its duration

$$\text{profit share} \propto \text{capital} \times \text{time}$$

Applies when the agreement allocates profit by capital-time exposure.

02 Split the investment timeline

$$W_i = \sum_j P_{ij} t_{ij}, \qquad S_i = \frac{W_i}{\sum W_i} S$$

Split each investor's timeline whenever their capital changes.

03 Convert a ratio into a total share

$$a : b = m : n \Rightarrow \frac{a}{a+b} = \frac{m}{m+n}$$

A part-to-part ratio becomes a part-to-total share using the sum of weights.

Quick method. Draw separate capital timelines and count actual months. Joining dates and investment changes can occur at different times.

EXAMPLE 1 **A invests 6000 for a year. B invests 9000 after four months. Divide profit 5000 by capital-time exposure.**

A contributes 6000 times 12; B contributes 9000 times 8. Their weights are equal.

$$S_A = S_B = 2500$$

EXAMPLE 2 **A starts with 8000 and withdraws 2000 after six months. B invests 6000 throughout the year. Divide profit 3900.**

A's weight is 8000 times 6 + 6000 times 6 = 84000; B's is 72000. Their ratio is 7:6.



Split the year where A withdraws capital.

A: 8000 x 6 + 6000 x 6 = 84000.

B: 6000 x 12 = 72000; ratio 7:6.

$$S_A = 2100, \qquad S_B = 1800$$

Watch for. A withdrawal changes future exposure; it does not remove capital from months already completed. A salary/commission agreement may require a deduction first.

Simple and Compound Interest

Simple interest, compound interest and instalments

01 Compare simple and compound interest

$$SI = \frac{Prt}{100}, \quad A = P\left(1 + \frac{r}{100}\right)^n$$

SI uses rate r percent per year; compound formula assumes n annual periods at a fixed rate.

02 Match the rate to the compounding period

$$A = P\left(1 + \frac{r}{100m}\right)^{mt}$$

Nominal annual r percent, compounded m times per year, for an integral number of compounding periods.

03 Use the two-year interest difference

$$CI - SI = P\left(\frac{r}{100}\right)^2$$

For exactly two years at the same annual rate and annual compounding.

Quick method. For changing rates, multiply the separate period factors. Compare instalments at the same date, accounting for the interest each has earned or avoided.

EXAMPLE 1 Find CI minus SI on 5000 at 10% annually for two years.

Use the two-year difference directly, or compare compound amount 6050 with simple amount 6000.

$$CI - SI = 5000(0.1)^2 = 50$$

EXAMPLE 2 A debt of 1210 is due one year from now. It will be settled by an equal payment now and another after one year at 10% annual compound interest. Find each payment.

At the due date, the first payment contributes 1.1x and the second x.

$$1.1x + x = 1210 \Rightarrow x = \frac{12100}{21}$$

Watch for. A half-yearly rate is half the nominal annual rate, but the number of periods doubles. Compound interest is amount minus principal.

Time, Distance and Races

Speed, relative motion and average speed

01 Keep distance, speed and time consistent

$$d = vt, \quad 1 \text{ km/h} = \frac{5}{18} \text{ m/s}$$

Use consistent distance and time units.

02 Find the relative speed

$$v_{\text{relative}} = u + v \text{ (opposite)}, \quad |u - v| \text{ (same direction)}$$

These forms assume straight-line motion along the same track.

03 Calculate a journey's average speed

$$v_{\text{avg}} = \frac{\text{total distance}}{\text{total time}}, \quad v_{\text{equal distances}} = \frac{2uv}{u+v}$$

The shortcut requires two equal-distance legs and positive speeds.

Quick method. For equal time intervals, speed averages arithmetically; for equal distances, use the harmonic form. In pursuit, divide the initial gap by the closing speed.

EXAMPLE 1 **A vehicle covers equal distances at 30 and 60 km/h. Find its average speed.**

Use the equal-distance formula; the slower leg consumes more time.

$$v = \frac{2 \cdot 30 \cdot 60}{30 + 60} = 40 \text{ km/h}$$

EXAMPLE 2 **A runner at 12 km/h chases another at 9 km/h who starts 150 m ahead. Find the catch-up time.**

The closing speed is 3 km/h. Convert the gap to 0.15 km.

$$t = \frac{0.15}{3} \text{ h} = 3 \text{ min}$$

Watch for. The arithmetic mean of speeds is not generally the average speed of a journey. Account for stops in the total elapsed time when required.

Time, Distance and Races

Trains, poles and platforms

01 Count the length that must clear

$$t_{\text{pole}} = \frac{L}{v}, \quad t_{\text{platform}} = \frac{L+P}{v}$$

Train length L, platform length P; convert speed to the matching length/time unit.

02 Cross another moving train

$$t_{\text{crossing}} = \frac{L_1 + L_2}{v_{\text{relative}}}$$

For two trains fully crossing each other.

03 Convert speed before using metre lengths

$$1 \text{ km/h} = 5/18 \text{ m/s}$$

Convert speeds before combining metre lengths with times in seconds.

Quick method. Use the whole length that must clear the obstacle. For crossing trains, add both lengths and use the speed relative to the other train.

EXAMPLE 1 **A train 180 m long crosses a 120 m platform at 54 km/h. Find the time.**

Convert 54 km/h to 15 m/s and use the combined 300 m length.

$$t = \frac{180 + 120}{15} = 20 \text{ s}$$

EXAMPLE 2 **Two trains 120 m and 180 m long travel in opposite directions at 36 and 54 km/h. Find their full crossing time.**

Combined length is 300 m. Relative speed is 90 km/h = 25 m/s.

$$t = 300/25 = 12 \text{ s}$$

Watch for. Passing a pole involves the train's length; passing a platform involves both lengths. Same-direction relative speeds subtract.

Time, Distance and Races

Boats, streams and round trips

01 Add or subtract the stream speed

$$v_d = u + v, \quad v_u = u - v$$

u is still-water speed; v is stream speed. Upstream motion requires u > v >= 0.

02 Recover still-water and stream speeds

$$u = (v_d + v_u)/2, \quad v = (v_d - v_u)/2$$

Downstream and upstream speeds must refer to the same boat and constant stream.

03 Add the two journey times

$$T = \frac{d}{u+v} + \frac{d}{u-v}$$

Round trip over the same distance d each way, with no stops.

Quick method. Separate actual speed from speed relative to the water. For an object simply drifting, ground speed equals stream speed.

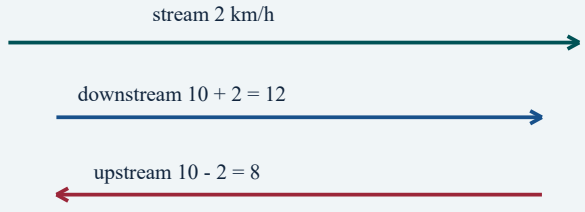
EXAMPLE 1 A boat moves at 15 km/h downstream and 9 km/h upstream. Find boat and stream speeds.

Half the sum gives the boat's still-water speed; half the difference gives the stream.

$$u = 12, \quad v = 3 \text{ km/h}$$

EXAMPLE 2 A boat moves at 10 km/h in still water in a 2 km/h stream. Find the time to travel 24 km downstream and return.

Downstream takes 24/12 = 2 hours; upstream takes 24/8 = 3 hours.



Boat speed in still water is 10 km/h.

Equal-distance legs take unequal times.

For 24 km each way: 2 h + 3 h = 5 h.

$$T = 2 + 3 = 5 \text{ h}$$

Watch for. Equal travel distances do not mean equal travel times. The arithmetic mean of upstream/downstream speeds is still-water speed, not round-trip average speed.

Time, Distance and Races

Races, head starts and circular tracks

01 Convert a race margin to a speed ratio

$$\text{speed ratio} = L : (L - d)$$

A beats B by d distance units in a race of length L, with simultaneous starts and constant speeds.

02 Close a head-start gap

$$t_{\text{catch}} = g/(u - v), \quad u > v$$

A faster runner of speed u pursues a runner of speed v with gap g on a straight track.

03 Distinguish a lap from a meeting

$$t_{\text{lap}} = L/|u - v|, \quad t_{\text{meet}} = L/(u + v)$$

Same/opposite directions on one circular track of length L, starting together; unequal speeds for the lap formula.

Quick method. Compare distances travelled in the same elapsed time. Convert a time head start into a distance gap before using relative speed.

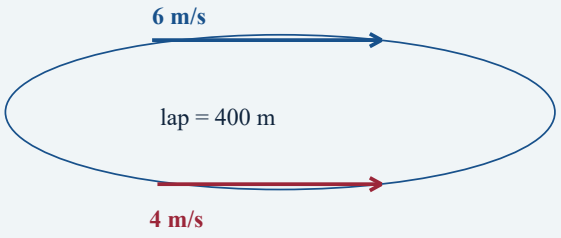
EXAMPLE 1 **A beats B by 20 m in a 100 m race. At the same speeds, by how much does A beat B in a 200 m race ?**

Their speed ratio is 100:80 = 5:4. When A runs 200 m, B runs 160 m.

$$\text{margin} = 200 - 160 = 40 \text{ m}$$

EXAMPLE 2 **Two runners start together on a 400 m circular track at 6 and 4 m/s in the same direction. When does the faster runner first overtake the slower ?**

The faster must gain one full lap at a relative speed of 2 m/s.



Both runners travel in the same direction.

One overtake requires gaining one lap.

Closing speed is 6 - 4 = 2 m/s.

$$t = 400/(6 - 4) = 200 \text{ s}$$

Watch for. A distance handicap and a time handicap are different. On a circular track, meeting again is not necessarily returning together to the starting line.

Area and Perimeter

Plane areas, perimeters and sectors

01 Use a perpendicular height

$$A_{\text{triangle}} = \tfrac{1}{2}bh, \quad A_{\text{trapezium}} = \tfrac{1}{2}(a+b)h$$

a,b are parallel sides of the trapezium; h is perpendicular separation.

02 Separate circumference, area and sector

$$C = 2\pi r, \quad A_{\text{circle}} = \pi r^2, \quad A_{\text{sector}} = \frac{\theta}{360^\circ}\pi r^2$$

Theta is the central angle in degrees.

03 Recognise special triangle areas

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}, \quad A_{\text{equilateral}} = \frac{\sqrt{3}}{4}a^2$$

Heron's semiperimeter s = (a + b + c)/2; use nondegenerate triangle sides.

Quick method. For a semicircle, add its diameter to the curved arc when finding total perimeter. For a ring or path, subtract the inner area from the outer area.

EXAMPLE 1 **Find the total perimeter of a semicircle of radius 7 cm, using pi = 22/7.**

Add the half-circumference and the diameter.

$$P = \pi r + 2r = 22 + 14 = 36 \text{ cm}$$

EXAMPLE 2 **Find the area of a triangle with sides 13,14,15 cm.**

s = 21; insert into Heron's formula.

$$\Delta = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84 \text{ cm}^2$$

Watch for. Perimeter scales with length; area with length squared. Do not use a slant side as a perpendicular height.

Area and Perimeter

Composite areas, paths and scaling

01 Subtract inner area from outer area

$$A_{\text{outer path}} = (l + 2w)(b + 2w) - lb$$

A uniform path of width w surrounds a rectangular region of dimensions l by b.

02 Calculate an annular region

$$A_{\text{ring}} = \pi(R^2 - r^2)$$

R and r are outer/inner radii, with R > r >= 0.

03 Square the linear scale factor

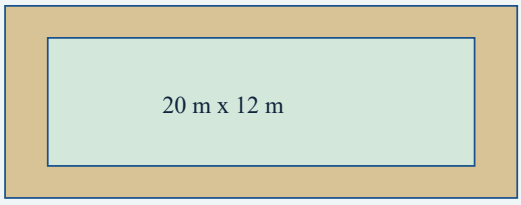
$$\text{length factor } k \Rightarrow \text{area factor } k^2$$

All corresponding lengths must scale by k; perimeter then scales by k too.

Quick method. Split a composite region into known shapes or subtract the unwanted region. For equal perimeters, derive each shape's side before comparing areas.

EXAMPLE 1 A 20 m by 12 m garden has a 2 m wide path outside it. Find the path area.

Both overall dimensions increase by twice the path width.



The path adds 2w to each dimension.

Subtract garden area from outer area.

The shaded path includes the corners.

$$A = 24 \cdot 16 - 20 \cdot 12 = 144 \text{ m}^2$$

EXAMPLE 2 Each side of a square increases by 20%. Find the percentage rise in its area.

Area scales by the square of the side factor, not by the factor itself.

$$1.2^2 = 1.44 \Rightarrow 44\% \text{ increase}$$

Watch for. A parallelogram's adjacent side is not generally its height. Overlapping paths or cut-outs must not be counted twice.

Probability

Probability, complements and overlapping events

01 Start with equally likely outcomes

$$P(A) = \frac{n(A)}{n(S)}, \quad 0 \le P(A) \le 1$$

The counting form assumes a finite set of equally likely elementary outcomes.

02 Use the complementary event

$$P(A^c) = 1 - P(A)$$

At least one is often easier through the complement none.

03 Subtract overlapping outcomes once

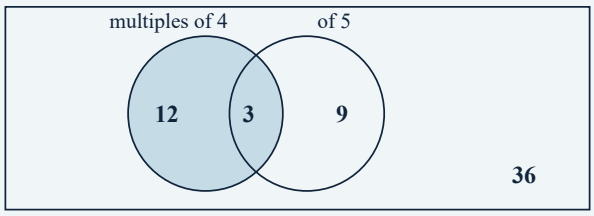
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Subtract the overlap once. Mutually exclusive events have zero intersection.

Quick method. Write what one outcome means: a card, an integer, or an ordered pair. Count favourable outcomes using the same sample space.

EXAMPLE 1 **Choose an integer uniformly from 1 to 60. Find the probability of a multiple of 4 or 5.**

Count 15 multiples of 4 and 12 of 5, subtracting the 3 multiples of 20 counted twice.



Sample space: integers 1 to 60.

Intersection: multiples of 20, count 3.

Union count = 12 + 3 + 9 = 24.

$$P = \frac{15 + 12 - 3}{60} = \frac{2}{5}$$

EXAMPLE 2 **Two fair independent dice are thrown. Find the probability of at least one six.**

The complement has neither die equal to six, probability (5/6) squared.

$$P = 1 - (5/6)^2 = \frac{11}{36}$$

Watch for. Either/or usually includes the overlap unless the wording says exactly one. Equally many categories do not imply equally likely outcomes.

Probability

Successive draws and independence

01 Use the probability after the first event

$$P(A \cap B) = P(A)P(B \mid A)$$

This conditional form requires P(A) > 0; use the second-event probability after the first event.

02 Recognise independence

$$P(A \cap B) = P(A)P(B) \quad \text{if independent}$$

Independent does not mean mutually exclusive; replacement may restore independence in standard draws.

03 Update counts without replacement

$$P(\text{two red}) = \frac{R}{N}\frac{R-1}{N-1}$$

Uniform successive draws without replacement from N balls, R red; N >= 2.

Quick method. For one of each colour, add both orders. Without replacement, update both the total count and the affected colour count after the first draw.

EXAMPLE 1 A bag contains four red and six blue balls. Two are drawn without replacement. Find the probability of one of each colour.

Add red-then-blue and blue-then-red.



$$P = \frac{4}{10}\frac{6}{9} + \frac{6}{10}\frac{4}{9} = \frac{8}{15}$$

EXAMPLE 2 The same bag is used with replacement after the first draw. Find the probability that both balls are red.

Replacement restores the original 4/10 probability for the independent second draw.

$$P = (4/10)^2 = \frac{4}{25}$$

Watch for. Do not retain the original denominator on the second draw without replacement. State whether order matters before counting combinations.

Number System & Number Series

Number system, digits and divisibility

01 Use decimal place value

$$N = 10a + b, \quad N_{\text{reversed}} = 10b + a$$

For a two-digit number, 1 <= a <= 9 and 0 <= b <= 9.

02 Apply digit-sum and alternating-sum tests

$$10a + b \equiv a + b \pmod 9, \quad 10a + b \equiv b - a \pmod{11}$$

Digit-sum and alternating-digit-sum tests extend to longer integers.

03 Write the division algorithm

$$a = bq + r, \quad 0 \le r < b$$

Division algorithm for an integer a and positive integer divisor b.

Quick method. Use digit variables only after fixing place value. Divisibility by 6 requires both 2 and 3; by 12 requires both 3 and 4. An irrational number cannot be written as a ratio of integers.

EXAMPLE 1 Find the digit x for which 47x2 is divisible by both 9 and 4.

The digit sum is 13 + x, so divisibility by 9 forces x = 5. The final two digits are then 52, divisible by 4.

$$x = 5, \quad 4752 \div 36 = 132$$

EXAMPLE 2 A two-digit number has digit sum 9 and exceeds its reversal by 27. Find the number.

Write a + b = 9 and 9(a - b) = 27. Solving gives a = 6, b = 3.

$$N = 63$$

Watch for. A leading digit cannot be zero. Divisibility by 4 uses the final two digits; by 8 uses the final three, not the digit sum.

Number System & Number Series

Number series and finite differences

01 Recognise constant differences

$$a_n = a + (n-1)d, \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

Arithmetic sequence: constant difference d; n is a positive integer.

02 Recognise a constant ratio

$$a_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r - 1}$$

Geometric sequence: constant ratio r; this sum form requires r not equal to 1.

03 Inspect first and second differences

$$\Delta a_n = a_{n+1} - a_n, \quad \Delta^2 a_n = \Delta a_{n+1} - \Delta a_n$$

Constant second differences suggest a quadratic sequence.

Quick method. Try differences, ratios, squares/cubes, alternating subsequences and multiply-then-add rules. Validate a proposed rule against every displayed transition.

EXAMPLE 1 **Find the next term in 5, 11, 19, 29, 41, ... using the simplest difference pattern.**

The differences are 6,8,10,12; continue with 14.

5	11	19	29	41	55
	+6	+8	+10	+12	+14

Differences increase by 2 each time.

The first differences are not constant.

The second differences are constant.

Check every step before extending.

$$41 + 14 = 55$$

EXAMPLE 2 **Find the sum of the first eight terms of 3,6,12,24, ...**

This is geometric with first term 3 and ratio 2.

$$S_8 = 3(2^8 - 1) = 765$$

Watch for. A short finite list admits many mathematical continuations. In an exam, choose the simplest consistent intended pattern supported by the options.

Mixture & Allegation

Mixtures and alligation

01 Calculate a weighted concentration

$$c = \frac{q_1 c_1 + q_2 c_2}{q_1 + q_2}$$

Concentrations or prices must use the same basis; quantities must be additive in the stated model.

02 Use opposite concentration differences

$$q_1 : q_2 = (c_2 - c) : (c - c_1)$$

For c1 < c < c2; opposite differences give the low-to-high quantity ratio.

03 Find the high-strength fraction

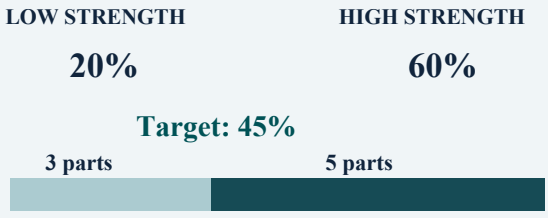
$$f_{\text{high}} = \frac{c - c_1}{c_2 - c_1}$$

The high-concentration share is a fraction of the total mixture, not a ratio to the low share.

Quick method. Combine equal-sized groups into their average concentration first, then apply alligation against a third solution.

EXAMPLE 1 How much 60% solution is needed in 120 litres of a 45% mixture made from 20% and 60% solutions ?

The high-strength fraction is (45 - 20)/(60 - 20) = 5/8.



Low quantity : high quantity = 3 : 5

Low parts: 60 - 45 = 15.

High parts: 45 - 20 = 25; reduce 15:25.

$$q_{\text{high}} = 120 \cdot \frac{5}{8} = 75 \text{ L}$$

EXAMPLE 2 Equal amounts of 20% and 40% solutions are combined, then mixed with 70% solution to obtain 50%. Find the ratio of the combined first two to the third.

The equal first pair averages 30%. Alligate 30% and 70% around 50%.

$$q_{\text{pair}} : q_3 = 20 : 20 = 1 : 1$$

Watch for. A mixture cannot lie outside the range of its component concentrations. Do not average different-sized batches equally.

Mixture & Allegation

Dilution and repeated replacement

01 Track what remains after one refill

$$C_1 = C_0(1 - v/V)$$

Remove v from a well-mixed container of volume V, then refill with pure diluent.

02 Repeat the retained fraction

$$C_n = C_0(1 - v/V)^n$$

Repeat the same removal/refill step n times, mixing before each withdrawal.

03 Find a single replacement fraction

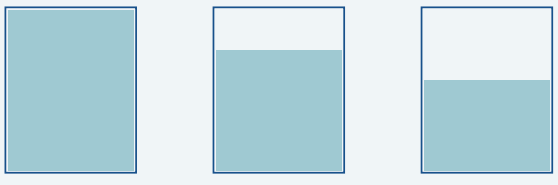
$$v/V = 1 - C_1/C_0$$

For one replacement with pure diluent, using nonzero initial concentration C0.

Quick method. Track the component that the refill does not contain. In milk-water problems, the retained milk fraction multiplies at every replacement.

EXAMPLE 1 A 40-litre container holds pure juice. Ten litres are removed and replaced by water, twice. How much juice remains ?

Each operation retains 30/40 = 3/4 of the juice present immediately before it.



Track juice, not total container volume.

Each step retains 3/4 of existing juice.

The container is refilled and mixed.

$$J = 40(3/4)^2 = 22.5 \text{ L}$$

EXAMPLE 2 Milk and water are in ratio 3:2. What fraction of the mixture must be replaced by water to make the ratio 2:3 ?

Milk concentration must fall from 3/5 to 2/5. Its retained fraction is 2/3.

$$v/V = 1 - \frac{2/5}{3/5} = \frac{1}{3}$$

Watch for. Two withdrawals of v do not remove the same amount of solute: the second mixture is weaker. The fixed-volume formula needs refilling after every withdrawal.

Square & Square root / Cube & Cube roots

Squares, cubes and their roots

01 Distinguish square roots from cube roots

$$\sqrt{a^2} = |a|, \quad \sqrt[3]{a^3} = a$$

Real cube roots preserve the sign; principal square roots are nonnegative.

02 Check prime-exponent groups

$$N = \prod p_i^{e_i} : \quad \text{square if } 2 \mid e_i; \quad \text{cube if } 3 \mid e_i$$

For positive integers, every prime exponent must satisfy the indicated condition.

03 Match a surd's sum and product

$$(\sqrt{u} + \sqrt{v})^2 = u + v + 2\sqrt{uv}$$

For nonnegative u,v, denest by matching the sum and product.

Quick method. For the least multiplier making a square or cube, supply only the missing prime powers. Memorise small squares and cubes to avoid trial division.

EXAMPLE 1 **Find the smallest positive integer by which 72 must be multiplied to become a perfect cube.**

72 = 2 cubed times 3 squared; supply one factor of 3.

2 x 2 x 2	complete triple	**Group prime factors in triples for cubes.**
3 x 3 x [3]	one 3 missing	72 has one full group of twos.
		One extra factor 3 completes the cube.

$$72 \cdot 3 = 216 = 6^3$$

EXAMPLE 2 **Simplify square root of (9 + 4 square root of 5).**

Seek u + v = 9 and uv = 20. Choose 4 and 5, and take their positive root sum.

$$\sqrt{9 + 4\sqrt{5}} = 2 + \sqrt{5}$$

Watch for. The cube root of a sum is not the sum of cube roots. Check the sign when choosing between conjugate surd forms.

HCF & LCM

Prime factors, HCF, LCM and Euclid

01 Count positive divisors

$$N = p_1^{a_1} \cdots p_k^{a_k} \Rightarrow d(N) = \prod_{i=1}^{k} (a_i + 1)$$

Distinct primes p_i; d(N) counts positive divisors, including 1 and N.

02 Link the HCF and LCM of two numbers

$$\gcd(a, b)\,\mathrm{lcm}(a, b) = ab$$

Positive integers a,b; this two-number product identity does not extend directly to three numbers.

03 Reduce the pair using Euclid

$$\gcd(a, b) = \gcd(b, a \bmod b)$$

Apply the Euclidean algorithm repeatedly until the remainder is zero.

Quick method. For prime factorizations, HCF takes the smaller exponents and LCM the larger. A prime has exactly two positive divisors; 1 is neither prime nor composite.

EXAMPLE 1 Find the HCF and LCM of 84 and 126.

The Euclidean steps leave 42 then zero. Use the two-number product identity for the LCM.

$$\gcd = 42, \quad \mathrm{lcm} = \frac{84 \cdot 126}{42} = 252$$

EXAMPLE 2 How many positive divisors does 360 have ?

Factor it as 2 cubed times 3 squared times 5. Each prime exponent can be selected independently.

$$d(360) = (3 + 1)(2 + 1)(1 + 1) = 24$$

Watch for. A smallest common multiple satisfying extra remainder conditions may need a final offset; do not stop at the LCM automatically.

Elementary Mensuration

Cubes, cuboids and space diagonals

01 Separate volume from total surface area

$$V = lbh, \quad TSA = 2(lb + bh + hl)$$

A closed rectangular cuboid; open boxes omit the missing faces.

02 Find the internal space diagonal

$$d = \sqrt{l^2 + b^2 + h^2}, \quad V_{\text{cube}} = a^3$$

d is the cuboid's internal space diagonal.

03 Identify the surfaces being counted

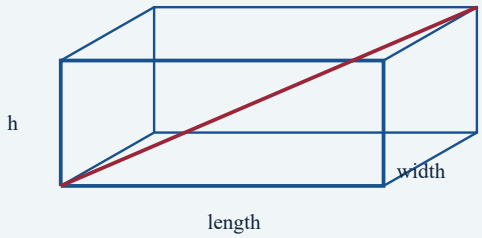
$$TSA_{\text{cube}} = 6a^2, \quad LSA_{\text{cuboid}} = 2h(l + b)$$

Lateral area omits the top and bottom for height h.

Quick method. List which faces exist before calculating surface area. For a recast solid, equate volumes if material is conserved; surface area generally changes.

EXAMPLE 1 **Find the space diagonal of a cuboid of dimensions 3,4,12 cm.**

Apply the three-dimensional Pythagorean formula.



The space diagonal crosses the interior.

Its square is l squared + b squared + h squared.

Open and closed boxes have different areas.

$$d = \sqrt{9 + 16 + 144} = 13 \text{ cm}$$

EXAMPLE 2 **A closed cube has total surface area 216 square cm. Find its volume.**

6a squared = 216 gives a = 6 cm; then cube the edge.

$$V = 6^3 = 216 \text{ cm}^3$$

Watch for. A space diagonal differs from a face diagonal. Capacity in litres is not the same numerical unit as cubic centimetres.

Elementary Mensuration

Cylinders, hollow pipes and capacity

01 Use the cylinder's radius and height

$$V = \pi r^2 h, \quad CSA = 2\pi rh, \quad TSA = 2\pi r(h + r)$$

For a right circular cylinder; TSA includes both circular ends.

02 Subtract the cylindrical void

$$V_{\text{material}} = \pi(R^2 - r^2)h$$

Hollow cylinder with outer radius R, inner radius r and R > r.

03 Convert capacity and cubic units

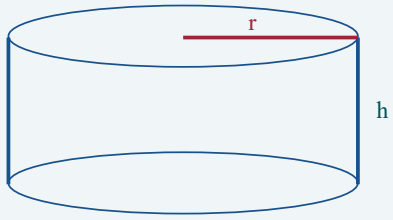
$$1\ \text{L} = 1000\ \text{cm}^3, \quad 1\ \text{m}^3 = 1000\ \text{L}$$

Convert all linear measurements into a common unit before cubing them.

Quick method. An open cylindrical tank's area is the curved side plus only its base. A pipe may require internal, external, end-ring or combined area; identify the requested surface.

EXAMPLE 1 A cylinder has radius 7 cm and height 10 cm. Find its volume using pi = 22/7.

Insert radius and height in pi r squared h.



Volume = base area times perpendicular height.

Curved area unwraps into a rectangle.

Closed cylinder: add both circular ends.

$$V = 1540\ \text{cm}^3 = 1.54\ \text{L}$$

EXAMPLE 2 Find the material volume of a hollow cylinder with outer radius 5, inner radius 3 and length 10 cm.

Subtract the inner cylindrical void from the outer cylinder.

$$V = \pi(25 - 9)(10) = 160\pi\ \text{cm}^3$$

Watch for. Changing diameter changes radius by the same factor, but volume at fixed height changes by its square.

Elementary Mensuration

Cones, spheres and volume conservation

01 Distinguish slant and perpendicular heights

$$V_{\text{cone}} = \tfrac{1}{3}\pi r^2 h, \quad l = \sqrt{r^2 + h^2}, \quad CSA = \pi r l$$

Right circular cone: h is perpendicular height, l slant height.

02 Use sphere and hemisphere relationships

$$V_{\text{sphere}} = \tfrac{4}{3}\pi r^3, \quad A_{\text{sphere}} = 4\pi r^2$$

A hemisphere has half the sphere's volume and curved area; add its base for total area.

03 Count complete units after material loss

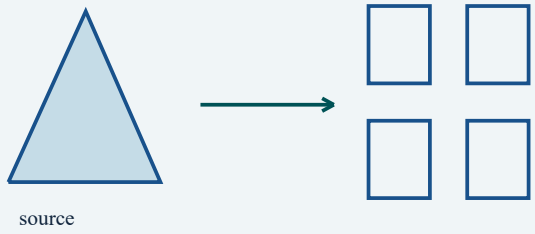
$$n = \left\lfloor \frac{(1-f)V_{\text{source}}}{V_{\text{unit}}} \right\rfloor$$

For the number of whole units recast/poured with fractional loss f, 0 <= f < 1.

Quick method. Use volume conservation when material is recast or liquid is transferred. Cancel common pi and square factors before doing arithmetic.

EXAMPLE 1 **A cone of radius 6 cm and height 12 cm is full of water. How many cylindrical cups of radius 3 cm and height 4 cm can it completely fill ?**

Divide cone volume by cup volume; pi cancels.



Compare volumes, not surface areas.

Cancel common pi before calculating.

Only completely filled cups count.

$$n = \frac{(1/3)6^2 \cdot 12}{3^2 \cdot 4} = 4$$

EXAMPLE 2 **A cuboid 12 cm by 10 cm by 5 cm is recast into cubes of side 2 cm. With 10% material loss, how many complete cubes result ?**

Usable volume is 540 cubic cm; each cube needs 8 cubic cm.

$$n = \lfloor 540/8 \rfloor = 67$$

Watch for. A cone's total area includes its base; a hemisphere's does too. Whole-unit counts round down, even when the leftover exceeds half a unit.

Heights and Distance

Heights, distances and two observation points

01 Relate height to horizontal distance

$$h = d\tan\theta$$

Height h above eye level, horizontal separation d and elevation theta.

02 Use a same-side two-point setup

$$h = \frac{d\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

Same-side ground-level points separated by d, nearer elevation beta > farther elevation alpha.

03 Equate expressions for the same height

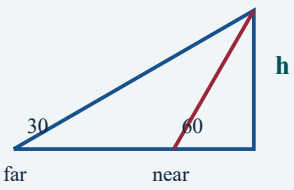
$$h = x\tan\beta = (x+d)\tan\alpha$$

Two points on the same side of a tower, separated by d; nearer angle beta > farther angle alpha.

Quick method. Set the unknown nearer distance to x and use x + d for the farther point. Add observer height only after calculating the height above the observer's horizontal level.

EXAMPLE 1 **Two ground-level points on the same side of a tower are 20 m apart. Elevations are 30 degrees at the farther point and 60 degrees at the nearer. Find the height.**

If the nearer distance is x, then x times square root of 3 = (x + 20)/square root of 3, giving x = 10.



angles in degrees

Same side: far distance = near distance + d.

Write both expressions for the same height.

Opposite-side points need a different setup.

$$h = 10\sqrt{3}\ \text{m}$$

EXAMPLE 2 **An observer whose eye is 1.5 m above ground stands 30 m from a building. The top's elevation is 45 degrees. Find the building height.**

The height above eye level is 30 tan 45 degrees = 30 m. Add 1.5 m.

$$H = 31.5\ \text{m}$$

Watch for. Points on opposite sides require a sum of distances, not the same-side setup. An angle of depression uses a parallel horizontal line.

Heights and Distance

Elevation, depression and reflections

01 Measure angles from a horizontal

$$\tan\theta = \frac{\text{vertical difference}}{\text{horizontal distance}}$$

The angle is measured from a horizontal line, not from the vertical.

02 Separate direct and reflected sight lines

$$\tan\alpha = \frac{H-h}{x}, \quad \tan\beta = \frac{H+h}{x}$$

Observer height h; object height H > h; reflection in a horizontal water surface; common horizontal separation x.

03 Eliminate the horizontal distance

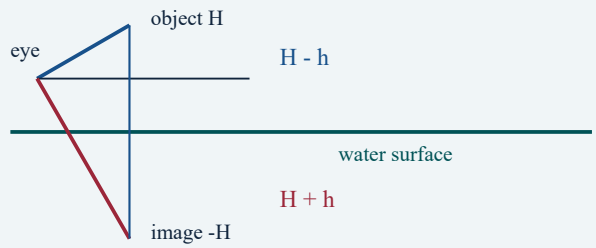
$$\frac{H}{h} = \frac{\tan\beta + \tan\alpha}{\tan\beta - \tan\alpha}$$

For the stated reflection geometry, beta > alpha; the reflection is H below water level.

Quick method. Draw the observer's horizontal first. Elevation to the object uses a height difference, while depression to its mirror image uses the sum.

EXAMPLE 1 **An observer is 20 m above water. A balloon has elevation 30 degrees and its reflection depression 60 degrees. Find the balloon's height.**

The tangents have ratio 1:3, so (H + 20)/(H - 20) = 3.



Direct ray uses vertical difference H - h.

Reflected ray uses vertical sum H + h.

Image and object are equally far from water.

$$H + 20 = 3H - 60 \Rightarrow H = 40 \text{ m}$$

EXAMPLE 2 **From the top of a 30 m tower, a ground vehicle is seen at depression 45 degrees. Find its horizontal distance from the foot.**

The angle of elevation at the vehicle is also 45 degrees; use the right triangle.

$$x = 30/\tan 45^\circ = 30 \text{ m}$$

Watch for. The reflected point is as far below the water as the object is above it. Do not use H instead of H - h for the direct sight line.

Statistics

Median, mode and data interpretation

01 Locate the middle value in sorted data

$$\text{Median} = x_{(n+1)/2} \quad (n \text{ odd})$$

Sort the observations in ascending order; the subscript is a position.

02 Average the two middle values

$$\text{Median} = \frac{x_{n/2} + x_{n/2+1}}{2} \quad (n \text{ even})$$

Use the two middle sorted observations, not the original input order.

03 Count the greatest frequency

$$\text{Mode} = \text{value(s) with highest frequency}$$

Data can have more than one mode; if every value occurs once, school conventions usually report no mode.

Quick method. After replacing values, sort again before locating the median. If the mean determines an unknown, solve the total first and then count frequencies.

EXAMPLE 1 **For 4,6,8,10,12,14 find the median. Then replace 6 by 20 and find the new median.**

Initially the middle pair is 8,10. After sorting again, it is 10,12.

4 6 8 10 12 14

Middle pair: (8 + 10)/2 = 9

Sort before choosing middle positions.

Even-sized data use two middle values.

Replacement may change their positions.

$$m_1 = 9, \quad m_2 = 11$$

EXAMPLE 2 **The mean of 5,7,x,x+2,11 is 8. Find x and the mode.**

The total is 40, so 25 + 2x = 40. The values are 5,7,7.5,9.5,11, all different.

$$x = 7.5; \quad \text{no mode}$$

Watch for. Mean, median and mode need not coincide. Do not force the empirical mean-mode-median relation onto arbitrary data.

Exponents and powers

Exponents, powers and efficient simplification

01 Combine powers of a common base

$$a^m a^n = a^{m+n}, \quad a^m/a^n = a^{m-n}, \quad (a^m)^n = a^{mn}$$

For unrestricted real exponents use positive bases; division needs a nonzero base.

02 Interpret zero and negative exponents

$$a^0 = 1, \quad a^{-n} = 1/a^n$$

a is nonzero. A negative exponent gives a reciprocal, not a negative value.

03 Factor before expanding

$$(a+b)^2 = a^2 + 2ab + b^2, \quad a^2 - b^2 = (a-b)(a+b)$$

Useful numerical identities; factor before expanding large expressions.

Quick method. Extract the smallest common power first. Convert all factors to one base where possible, then combine exponents.

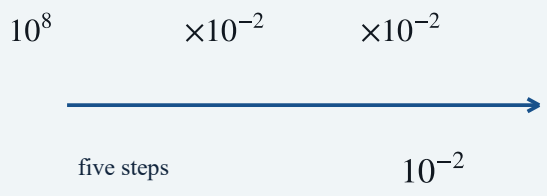
EXAMPLE 1 Simplify (3 to the power n+1 + 3 to the power n) divided by 3 to the power n-1.

Factor the numerator as 4 times 3 to the power n; one power of 3 remains after division.

$$\frac{3^{n+1} + 3^n}{3^{n-1}} = 4 \cdot 3 = 12$$

EXAMPLE 2 A signal starts at 10 to the power 8 units and is multiplied by 10 to the power -2 each minute. Find its strength after five minutes.

Multiply the decay factor five times; add the resulting exponents.



Repeated multiplication adds exponents.

Five decay factors contribute exponent -10.

Initial exponent 8 becomes 8 - 10 = -2.

$$10^8(10^{-2})^5 = 10^{-2} = 0.01$$

Watch for. Powers do not distribute over addition. For a real x, square root of x^2 is its absolute value.

Formula Sheet 1: Numbers and Calculations

Reduce calculation before multiplying. These anchors support mental work; they do not replace checking the question conditions.

Percentage fractions

$$50\% = \tfrac{1}{2}, \quad 25\% = \tfrac{1}{4}, \quad 20\% = \tfrac{1}{5}, \quad 12.5\% = \tfrac{1}{8}$$

More useful fractions

$$16\tfrac{2}{3}\% = \tfrac{1}{6}, \quad 33\tfrac{1}{3}\% = \tfrac{1}{3}, \quad 66\tfrac{2}{3}\% = \tfrac{2}{3}$$

Squares to recognise

$$11^2 = 121, \quad 12^2 = 144, \quad 13^2 = 169, \quad 14^2 = 196, \quad 15^2 = 225$$

Cubes to recognise

$$6^3 = 216, \quad 7^3 = 343, \quad 8^3 = 512, \quad 9^3 = 729, \quad 10^3 = 1000$$

Divisibility: 3 and 9

digit sum divisible by 3 or 9

Divisibility: 4, 8 and 11

$4:$ last 2 digits; $8:$ last 3; $11:$ alternating sum

HCF-LCM product

$$\gcd(a, b)\,\mathrm{lcm}(a, b) = ab \quad (a, b > 0 \text{ integers})$$

Perfect-power test

$N = \prod p_i^{e_i}$: $2 \mid e_i$ for squares; $3 \mid e_i$ for cubes

Difference of squares

$$99^2 = (100 - 1)^2 = 9801, \quad 51 \cdot 49 = 50^2 - 1 = 2499$$

Speed and capacity units

1 km/h $= 5/18$ m/s, 1 L $= 1000$ cm^3

Arithmetic and geometric terms

$$a_n = a + (n - 1)d, \quad b_n = ar^{n-1}$$

The alternating digit sum for divisibility by 11 must be a multiple of 11, including zero. For recurring decimals, reduce the fraction before testing its denominator for factors other than 2 and 5. Series patterns must fit all displayed terms.

Formula Sheet 2: Percentages and Rates

Every shortcut has a base or a rate assumption. Use it only after identifying those quantities.

Successive signed percentages

$$\Delta\% = p + q + pq/100$$

Markup and discount

$$SP = CP(1 + p/100)(1 - d/100)$$

Equal selling prices / equal gain-loss p

$$\text{overall loss}\% = p^2/100 \quad (0 < p < 100)$$

Two-year annual CI minus SI

$$CI - SI = P(r/100)^2$$

Recover the two-year rate from SI and CI

$$r = 200(CI - SI)/SI \quad (SI > 0)$$

Capital-time weights

$$W_i = \sum_j P_{ij} t_{ij}, \qquad S_i = S\frac{W_i}{\sum W_i}$$

Alligation

$$q_{\text{low}} : q_{\text{high}} = (c_{\text{high}} - c) : (c - c_{\text{low}})$$

Repeated replacement with diluent

$$C_n = C_0(1 - v/V)^n$$

Joint positive work rates

$$T_{AB} = ab/(a + b)$$

Equal-distance average speed

$$v_{\text{avg}} = 2uv/(u + v)$$

Same / opposite direction closing speed

$$|u - v| \quad / \quad u + v$$

Boat and stream from measured speeds

$$u = (v_d + v_u)/2, \qquad v = (v_d - v_u)/2$$

The recovered interest-rate rule assumes SI and CI on the same principal for exactly two years, one fixed annual percentage rate and annual compounding. The gain/loss shortcut needs equal selling prices on two items and equal gain/loss percentages. Replacement assumes a fixed volume mixed after every refill.

Formula Sheet 3: Areas, Volumes and Heights

Use perpendicular heights, radii rather than diameters, and only the surfaces that are actually exposed.

Rectangle / square / parallelogram

$$A = lb, \quad P = 2(l + b); \quad A_{\text{square}} = a^2; \quad A_{\text{parallelogram}} = bh$$

Triangle / trapezium

$$A = \tfrac{1}{2}bh, \quad A_{\text{trapezium}} = \tfrac{1}{2}(a + b)h$$

Equilateral triangle / regular hexagon

$$A_{\triangle} = \frac{\sqrt{3}}{4}a^2, \quad A_{\text{hexagon}} = \frac{3\sqrt{3}}{2}a^2$$

Circle / sector

$$A = \pi r^2, \quad C = 2\pi r, \quad A_{\text{sector}} = \frac{\theta}{360^\circ}\pi r^2$$

Cuboid / cube

$$V = lbh, \quad TSA = 2(lb + bh + hl), \quad d = \sqrt{l^2 + b^2 + h^2}$$

Cylinder

$$V = \pi r^2 h, \quad CSA = 2\pi rh, \quad TSA = 2\pi r(h + r)$$

Cone

$$V = \tfrac{1}{3}\pi r^2 h, \quad CSA = \pi rl, \quad TSA = \pi r(l + r)$$

Sphere / hemisphere

$$V_s = \tfrac{4}{3}\pi r^3, \quad A_s = 4\pi r^2, \quad V_h = \tfrac{2}{3}\pi r^3$$

Hemisphere surfaces

$$CSA = 2\pi r^2, \quad TSA = 3\pi r^2$$

Exact acute tangents

$$\tan 30^\circ = 1/\sqrt{3}, \quad \tan 45^\circ = 1, \quad \tan 60^\circ = \sqrt{3}$$

Right-triangle triples

$$(3, 4, 5), \quad (5, 12, 13), \quad (8, 15, 17), \quad (7, 24, 25)$$

For a parallelogram, h is the perpendicular height, not an arbitrary adjacent side. For sectors theta is in degrees. Cone slant height satisfies l squared = r squared + h squared. Similar lengths scale by k, areas by k squared and volumes by k cubed.

Formula Sheet 4: Statistics and Probability

Fix the sample space, the unit and the requested quantity before you calculate.

Mean / combined mean

$$\bar{x} = \frac{\sum x_i}{n}, \qquad \bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Weighted mean

$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}$$

Replacing a wrong observation

$$\bar{x}_{\text{new}} = \bar{x}_{\text{old}} + (\text{correct} - \text{wrong})/n$$

Median: sorted data

$$x_{(n+1)/2} \ (n \text{ odd}); \qquad (x_{n/2} + x_{n/2+1})/2 \ (n \text{ even})$$

Mode

value(s) with greatest frequency

Equally likely outcomes

$$P(A) = n(A)/n(S)$$

Complement / overlapping union

$$P(A^c) = 1 - P(A), \qquad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Successive events

$$P(A \cap B) = P(A)P(B \mid A)$$

Independent successive events

$$P(A \cap B) = P(A)P(B) \quad \text{only if independent}$$

Two red without replacement

$$P = \frac{R}{N}\frac{R-1}{N-1}$$

Feasibility bounds

$$0 \le P \le 1, \qquad \min x_i \le \bar{x} \le \max x_i$$

The conditional multiplication form needs P(A) > 0. Read whether draws replace the first object, whether order matters and whether either/or includes overlap. Check units, percentage bases, complete-unit rounding and plausible bounds. Estimate first; if the options are close, keep exact fractions until the end.