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Topic index

Number System

1 Number system, digits and divisibility 4
2 Remainders, powers and factorial zeros 5
3 Prime factors, HCF, LCM and Euclid 6
4 Common logarithms and decimal magnitude 7

Arithmetic

5 Fractions, decimal expansions and reciprocals 8
6 Indices, square roots and surds 9

Ratio and Proportion

7 Ratio, proportion and variation 10
8 Unitary method, mixtures and weighted shares 11

Percentage

9 Percentages and successive changes 12

Profit and Loss

10 Profit, loss, marked price and discount 13

Interest

11 Simple interest, compound interest and instalments 14

Time and Work

12 Time, work, efficiency and pipes 15

Speed and Distance

13 Speed, relative motion and average speed 16
14 Trains, platforms and boats 17

Topic index

Algebra

15 Identities, factorisation and remainder theorem 18
16 Polynomial HCF, LCM and rational expressions 19
17 Quadratic roots and parameter conditions 20
18 Simultaneous equations and word problems 21
19 Sets, conditional identities and graphical inequalities 22

Trigonometry

20 Ratios, exact values and complementary angles 23
21 Identities and efficient simplification 24
22 Heights, distances and two observation points 25

Geometry

23 Lines, parallel angles and polygons 26
24 Triangles: congruence, centres and inequalities 27
25 Similarity, proportional segments and area ratios 28
26 Parallelograms, rectangles, rhombi and squares 29
27 Circles, chords and cyclic angles 30
28 Tangents, secants and loci 31

Mensuration

29 Plane areas, perimeters and sectors 32
30 Cubes, cuboids and space diagonals 33
31 Cylinders, hollow pipes and capacity 34
32 Cones, slant height and similar sections 35
33 Spheres, hemispheres and recasting 36
34 Composite figures, field-book areas and exposed surfaces 37

Statistics

35 Frequencies, histograms, polygons and pie charts 38
36 Mean, median, mode and corrections 39

Arithmetic toolkit and percentage anchors 40
Algebra and exact trigonometric values 41
Geometry and mensuration at a glance 42
Number tests, data and final checks 43
Statement checks and data sufficiency 44

Number System

Number system, digits and divisibility

01 Use decimal place value

$$N = 10a + b, \quad N_{\text{reversed}} = 10b + a$$

For a two-digit number, 1 <= a <= 9 and 0 <= b <= 9.

02 Apply digit-sum and alternating-sum tests

$$10a + b \equiv a + b \pmod 9, \quad 10a + b \equiv b - a \pmod{11}$$

Digit-sum and alternating-digit-sum tests extend to longer integers.

03 Write the division algorithm

$$a = bq + r, \quad 0 \le r < b$$

Division algorithm for an integer a and positive integer divisor b.

Quick method. Use digit variables only after fixing place value. Divisibility by 6 requires both 2 and 3; by 12 requires both 3 and 4. An irrational number cannot be written as a ratio of integers.

EXAMPLE 1 **Find the digit x for which 47x2 is divisible by both 9 and 4.**

The digit sum is 13 + x, so divisibility by 9 forces x = 5. The final two digits are then 52, divisible by 4.

$$x = 5, \quad 4752 \div 36 = 132$$

EXAMPLE 2 **A two-digit number has digit sum 9 and exceeds its reversal by 27. Find the number.**

Write a + b = 9 and 9(a - b) = 27. Solving gives a = 6, b = 3.

$$N = 63$$

Watch for. A leading digit cannot be zero. Divisibility by 4 uses the final two digits; by 8 uses the final three, not the digit sum.

Number System

Remainders, powers and factorial zeros

01 Reduce powers through remainder cycles

$$a \equiv r \pmod{m} \Rightarrow a^k \equiv r^k \pmod{m}$$

For nonnegative integer k; reduce after multiplication to keep numbers small.

02 Subtract numbers with the same remainder

$$a \equiv b \pmod{m} \Rightarrow m \mid (a - b)$$

Same remainders imply a divisible difference.

03 Count trailing zeros in a factorial

$$Z(n!) = \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{25} \right\rfloor + \cdots$$

Decimal trailing zeros count factors of 5; factors of 2 are more plentiful.

Quick method. List the short remainder cycle, then reduce the exponent by its period. For decimal unit digits, use the cycle modulo 10.

EXAMPLE 1 **Find the remainder when 7 to the power 103 is divided by 5.**

Powers of 7 have the same residues as powers of 2 modulo 5, cycling 2,4,3,1. The exponent leaves remainder 3 modulo 4.

$$7^{103} \equiv 3 \pmod{5}$$

EXAMPLE 2 **How many trailing zeros occur in 100 factorial ?**

Count multiples of 5, then the additional factor in each multiple of 25.

$$Z = 20 + 4 = 24$$

Watch for. A zero exponent remainder in a cycle means its last term, not its first. Modular cancellation requires a coprime factor.

Number System

Prime factors, HCF, LCM and Euclid

01 Count positive divisors

$$N = p_1^{a_1} \cdots p_k^{a_k} \Rightarrow d(N) = \prod_{i=1}^{k} (a_i + 1)$$

Distinct primes p_i; d(N) counts positive divisors, including 1 and N.

02 Link the HCF and LCM of two numbers

$$\gcd(a, b)\,\mathrm{lcm}(a, b) = ab$$

Positive integers a,b; this two-number product identity does not extend directly to three numbers.

03 Reduce the pair using Euclid

$$\gcd(a, b) = \gcd(b, a \bmod b)$$

Apply the Euclidean algorithm repeatedly until the remainder is zero.

Quick method. For prime factorizations, HCF takes the smaller exponents and LCM the larger. A prime has exactly two positive divisors; 1 is neither prime nor composite.

EXAMPLE 1 **Find the HCF and LCM of 84 and 126.**

The Euclidean steps leave 42 then zero. Use the two-number product identity for the LCM.

$$\gcd = 42, \quad \mathrm{lcm} = \frac{84 \cdot 126}{42} = 252$$

EXAMPLE 2 **How many positive divisors does 360 have ?**

Factor it as 2 cubed times 3 squared times 5. Each prime exponent can be selected independently.

$$d(360) = (3 + 1)(2 + 1)(1 + 1) = 24$$

Watch for. A smallest common multiple satisfying extra remainder conditions may need a final offset; do not stop at the LCM automatically.

Number System

Common logarithms and decimal magnitude

01 Combine products and quotients inside logs

$$\log_{10}(ab) = \log_{10} a + \log_{10} b, \quad \log_{10}(a/b) = \log_{10} a - \log_{10} b$$

All logarithm arguments are positive.

02 Apply logarithms to powers

$$\log_{10}(a^n) = n\log_{10} a, \quad \log_{10} 10^k = k$$

Use exact power-of-ten values before approximations.

03 Locate a number between powers of ten

$$10^k \le N < 10^{k+1} \Rightarrow k \le \log_{10} N < k+1$$

For a positive integer N, its digit count is floor(log10 N) + 1.

Quick method. Separate a number into a power of 10 times a factor from 1 to 10. A log table gives a mantissa; decimal placement comes from the characteristic.

EXAMPLE 1 **Given log base 10 of 2 = 0.30103, find log base 10 of 0.008.**

Write 0.008 as 2 cubed divided by 1000. Apply the logarithm laws.

$$\log_{10}(0.008) = 3(0.30103) - 3 = -2.09691$$

EXAMPLE 2 **Given the same logarithm value, find the number of digits in 2 to the power 50.**

The logarithm is 50 times 0.30103 = 15.0515, lying between 15 and 16.

$$\lfloor 15.0515 \rfloor + 1 = 16$$

Watch for. The logarithm of a positive fraction below 1 is negative. log(a + b) is not log a + log b. The examination does not permit calculator use.

Arithmetic

Fractions, decimal expansions and reciprocals

01 Operate with fractions

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

Denominators and the divisor c/d must be nonzero.

02 Recognise a terminating decimal

$$\frac{p}{q} \text{ terminates} \Leftrightarrow q = 2^a 5^b$$

Reduce p/q to lowest terms with q > 0; a,b are nonnegative integers.

03 Convert a recurring decimal to a fraction

$$0.\overline{abc} = \frac{100a + 10b + c}{999}$$

$$0.a\overline{b} = \frac{10a + b - a}{90}$$

Here a, b and c are decimal digits. First numerator: the three-digit value 100a + 10b + c, not a product. Second formula: one nonrepeating and one repeating digit.

Quick method. Cross-cancel before multiplying fractions. For a recurring decimal, shift the repeating tail into alignment and subtract the two equations.

EXAMPLE 1 **Convert 0.1 followed by recurring 6 to a fraction.**

Let x = 0.1666... . Then 10x = 1.666... and 100x = 16.666...; subtraction removes the recurring tail.

$$90x = 15 \Rightarrow x = \frac{1}{6}$$

EXAMPLE 2 **Does 21/140 terminate, and what is its decimal value ?**

Reduce to 3/20. The denominator contains only 2 and 5.

$$\frac{21}{140} = \frac{3}{20} = 0.15$$

Watch for. The terminating-decimal test applies to the reduced denominator. A percentage is a fraction out of 100, not an extra unit to multiply twice.

Arithmetic

Indices, square roots and surds

01 Combine powers of a common base

$$a^{m+n} = a^m a^n, \quad a^{m-n} = \frac{a^m}{a^n}, \quad (a^m)^n = a^{mn}$$

Use a positive base for unrestricted real exponents; division needs a nonzero base.

02 Expand squares of surd sums

$$\sqrt{a^2} = |a|, \quad (\sqrt{a} \pm \sqrt{b})^2 = a + b \pm 2\sqrt{ab}$$

Real square roots require nonnegative radicands.

03 Rationalise a surd denominator

$$\frac{1}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$$

Rationalisation needs a nonzero original denominator and a not equal to b in this form.

Quick method. To denest square root of (a + 2 square root of b), seek positive u,v with u + v = a and uv = b. For a perfect square integer, every prime exponent is even.

EXAMPLE 1 **Simplify square root of (7 + 4 square root of 3).**

Look for u + v = 7 and uv = 12: choose 4 and 3. Their root sum is positive.

$$\sqrt{7 + 4\sqrt{3}} = 2 + \sqrt{3}$$

EXAMPLE 2 **Find the least positive integer multiplying 72 to produce a perfect square.**

72 = 2 cubed times 3 squared. Supply one more factor of 2.

$$72 \cdot 2 = 144 = 12^2$$

Watch for. Do not replace square root of x^2 by x when x may be negative. Roots do not distribute over addition.

Ratio and Proportion

Ratio, proportion and variation

01 Represent a ratio by one multiplier

$$a : b = m : n \Rightarrow a = km, \quad b = kn$$

A common multiplier preserves the ratio.

02 Distinguish direct and inverse variation

$$y \propto x \Rightarrow y = kx, \quad y \propto 1/x \Rightarrow xy = k$$

Direct and inverse variation have different invariants.

03 Compare compound variations

$$y \propto \frac{x^a}{z^b} \Rightarrow \frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^a \left(\frac{z_1}{z_2}\right)^b$$

Hold the proportionality constant fixed and use positive quantities where needed.

Quick method. Equalise the shared term when joining two ratios. In a work/production table, decide which variables vary directly and which inversely before multiplying ratios.

EXAMPLE 1 **A:B = 2:3 and B:C = 4:5. Find A:B:C.**

Make B equal to 12 in both ratios: multiply the first by 4 and the second by 3.

$$A : B : C = 8 : 12 : 15$$

EXAMPLE 2 **y varies directly as x^2 and inversely as z. It is 12 at x = 2, z = 3. Find y at x = 3, z = 6.**

Use the ratio of the two situations to avoid solving separately for the constant.

$$y = 12\left(\frac{3}{2}\right)^2 \frac{3}{6} = \frac{27}{2}$$

Watch for. Equal changes in two quantities do not preserve their ratio. Ratios need consistent units.

Ratio and Proportion

Unitary method, mixtures and weighted shares

01 Measure output in worker-hours

$$\text{output} = k\,\text{workers} \times \text{hours} \times \text{days}$$

Constant productivity, comparable workers and unchanged working conditions.

02 Use opposite concentration differences

$$\frac{q_1}{q_2} = \frac{c_2 - c}{c - c_1}$$

Alligation for c1 < c < c2; c is the mixture's weighted mean concentration/cost.

03 Weight capital by its duration

$$\text{profit share} \propto \text{capital} \times \text{time}$$

Partnership shares use agreed terms; this rule assumes shares follow capital-time exposure.

04 Repeat the retained concentration fraction

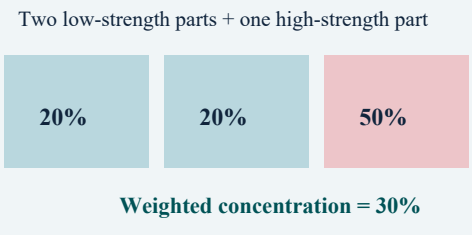
$$C_n = C_0\left(1 - \frac{v}{V}\right)^n$$

Replace v from a well-mixed fixed volume V with pure diluent each time.

Quick method. Choose a convenient common total and use weighted amounts. In replacement questions, mix thoroughly before every withdrawal; in partnerships, split time intervals if capital changes.

EXAMPLE 1 **In what ratio should 20% and 50% solutions be mixed to make a 30% solution ?**

Use opposite concentration differences: low-to-high quantity ratio is (50 - 30):(30 - 20).



Quantity ratio is 2:1, not 1:2.

Opposite differences give the quantities.

The average must lie between 20% and 50%.

$$q_1 : q_2 = 20 : 10 = 2 : 1$$

EXAMPLE 2 **A invests 5000 for 12 months and B 8000 for 9 months. Divide a profit of 2200.**

Capital-time weights are 60000:72000 = 5:6. Divide the profit into 11 equal parts.

$$A = 1000, \quad B = 1200$$

Watch for. Alligation uses differences in reverse order. Do not average concentrations without their volumes.

Percentage

Percentages and successive changes

01 Apply a signed percentage change

$$\text{new} = \text{old}\left(1 + \frac{p}{100}\right)$$

For a p percent decrease, replace +p by -p.

02 Combine successive changes

$$p \oplus q = p + q + \frac{pq}{100}$$

Net percentage change for successive signed changes p and q.

03 Recover the original quantity

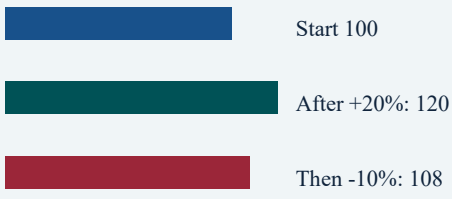
$$\text{original} = \frac{\text{final}}{1 + p/100}$$

The denominator is the total multiplying factor and must be nonzero; a complete 100 percent reduction cannot be reversed this way.

Quick method. Use fractions for common percentages: 12.5% = 1/8, 16 2/3% = 1/6, 25% = 1/4. Identify the base quantity before every comparison.

EXAMPLE 1 **A price rises by 20% and then falls by 10%. Find the overall change.**

Multiply 1.2 by 0.9, rather than subtracting the percentages.



Each percentage uses its current base.

A 10% drop is 12 after the rise to 120.

The overall increase is 8, or 8%.

$$1.2 \cdot 0.9 = 1.08 \Rightarrow 8\% \text{ increase}$$

EXAMPLE 2 **A value becomes 720 after a 10% reduction. Find its original value.**

The final value is 90% of the original; divide by 0.9.

$$V = 720 \div 0.9 = 800$$

Watch for. An increase of p percent followed by a decrease of p percent gives a net loss of p squared/100 percent, not zero.

Profit and Loss

Profit, loss, marked price and discount

01 Use cost price as the profit base

$$\text{profit}\% = 100\frac{SP - CP}{CP}, \quad SP = CP\left(1 + \frac{p}{100}\right)$$

Profit/loss percentage is based on cost price.

02 Apply a marked-price discount

$$SP = MP\left(1 - \frac{d}{100}\right)$$

Discount percentage is based on marked price.

03 Combine successive discounts

$$d_{\text{equivalent}} = d_1 + d_2 - \frac{d_1 d_2}{100}$$

For two successive discounts on the reducing marked amount.

Quick method. Track CP, MP and SP separately. In dishonest-weight questions, compare the amount paid for with the amount actually supplied.

EXAMPLE 1 **An article costing 800 is marked 25% above cost and sold at a 10% discount. Find the profit percentage.**

The marked price is 1000, and selling price is 900. The gain is 100 on a cost of 800.

$$\text{profit}\% = \frac{100}{800} \cdot 100 = 12.5\%$$

EXAMPLE 2 **Two articles are each sold for 1200, one at 20% profit and the other at 20% loss. Find the overall result.**

Their costs are 1000 and 1500. Total selling price is 2400 against total cost 2500.

$$\text{loss}\% = \frac{100}{2500} \cdot 100 = 4\%$$

Watch for. Equal profit and loss percentages on two items sold at the same price do not cancel: the cost prices differ.

Interest

Simple interest, compound interest and instalments

01 Compare simple and compound interest

$$SI = \frac{Prt}{100}, \quad A = P\left(1 + \frac{r}{100}\right)^n$$

SI uses rate r percent per year; compound formula assumes n annual periods at a fixed rate.

02 Match the rate to the compounding period

$$A = P\left(1 + \frac{r}{100m}\right)^{mt}$$

Nominal annual r percent, compounded m times per year, for an integral number of compounding periods.

03 Use the two-year interest difference

$$CI - SI = P\left(\frac{r}{100}\right)^2$$

For exactly two years at the same annual rate and annual compounding.

Quick method. For changing rates, multiply the separate period factors. Compare instalments at the same date, accounting for the interest each has earned or avoided.

EXAMPLE 1 **Find CI minus SI on 5000 at 10% annually for two years.**

Use the two-year difference directly, or compare compound amount 6050 with simple amount 6000.

$$CI - SI = 5000(0.1)^2 = 50$$

EXAMPLE 2 **A debt of 1210 is due one year from now. It will be settled by an equal payment now and another after one year at 10% annual compound interest. Find each payment.**

At the due date, the first payment contributes 1.1x and the second x.

$$1.1x + x = 1210 \Rightarrow x = \frac{12100}{21}$$

Watch for. A half-yearly rate is half the nominal annual rate, but the number of periods doubles. Compound interest is amount minus principal.

Time and Work

Time, work, efficiency and pipes

01 Think in rates

$$\text{rate} = \frac{1}{\text{time}}, \qquad \text{work} = \text{rate} \times \text{time}$$

For total work normalised to 1, rate = 1/time; otherwise use work/time. Add constant rates only while workers/pipes act together.

02 Combine two positive work rates

$$T_{AB} = \frac{ab}{a+b}$$

A and B separately finish the same work in a and b positive time units.

03 Relate efficiency to completion time

$$\text{efficiency ratio } m : n \Rightarrow \text{time ratio } n : m$$

For the same total work; an outlet or destroyer has a negative contribution.

Quick method. Choose the LCM of individual times as the total work, making hourly or daily rates integers. For changing teams, compute completed work stage by stage.

EXAMPLE 1 **A can finish a job in 12 days and B in 18 days. How long do they take together ?**

Take total work as 36 units. Their daily rates are 3 and 2 units, totalling 5.



AFTER ONE DAY A: 3 units + B: 2 units = 5 of 36 units complete

$$T = \frac{36}{5} = 7.2 \text{ days}$$

EXAMPLE 2 **A pipe fills a tank in 6 hours while a leak empties a full tank in 9 hours. How long to fill an empty tank with both acting ?**

Subtract the leak rate from the inlet rate, assuming both rates are constant.

$$\text{net rate} = \frac{1}{6} - \frac{1}{9} = \frac{1}{18} \Rightarrow T = 18 \text{ h}$$

Watch for. Add rates, not completion times. An outlet can make the net rate zero or negative, in which case the tank will not fill under those conditions.

Speed and Distance

Speed, relative motion and average speed

01 Keep distance, speed and time consistent

$$d = vt, \quad 1 \text{ km/h} = \frac{5}{18} \text{ m/s}$$

Use consistent distance and time units.

02 Find the relative speed

$$v_{\text{relative}} = u + v \text{ (opposite)}, \quad |u - v| \text{ (same direction)}$$

These forms assume straight-line motion along the same track.

03 Calculate a journey's average speed

$$v_{\text{avg}} = \frac{\text{total distance}}{\text{total time}}, \quad v_{\text{equal distances}} = \frac{2uv}{u+v}$$

The shortcut requires two equal-distance legs and positive speeds.

Quick method. For equal time intervals, speed averages arithmetically; for equal distances, use the harmonic form. In pursuit, divide the initial gap by the closing speed.

EXAMPLE 1 **A vehicle covers equal distances at 30 and 60 km/h. Find its average speed.**

Use the equal-distance formula; the slower leg consumes more time.

$$v = \frac{2 \cdot 30 \cdot 60}{30 + 60} = 40 \text{ km/h}$$

EXAMPLE 2 **A runner at 12 km/h chases another at 9 km/h who starts 150 m ahead. Find the catch-up time.**

The closing speed is 3 km/h. Convert the gap to 0.15 km.

$$t = \frac{0.15}{3} \text{ h} = 3 \text{ min}$$

Watch for. The arithmetic mean of speeds is not generally the average speed of a journey. Account for stops in the total elapsed time when required.

Speed and Distance

Trains, platforms and boats

01 Count the length that must clear

$$t_{\text{pole}} = \frac{L}{v}, \quad t_{\text{platform}} = \frac{L+P}{v}$$

Train length L, platform length P; convert speed to the matching length/time unit.

02 Cross another moving train

$$t_{\text{crossing}} = \frac{L_1 + L_2}{v_{\text{relative}}}$$

For two trains fully crossing each other.

03 Add or subtract the stream speed

$$v_{\text{down}} = u + v, \quad v_{\text{up}} = u - v$$

u is still-water boat speed; v is stream speed; upstream travel requires u > v.

Quick method. For boats, half the sum of downstream and upstream speeds gives still-water speed; half their difference gives stream speed.

EXAMPLE 1 **A train 180 m long crosses a 120 m platform at 54 km/h. Find the time.**

Convert 54 km/h to 15 m/s and use the combined 300 m length.

$$t = \frac{180 + 120}{15} = 20 \text{ s}$$

EXAMPLE 2 **A boat travels at 18 km/h downstream and 10 km/h upstream. Find boat and stream speeds.**

Take half the sum and half the difference.

$$u = 14, \quad v = 4 \text{ km/h}$$

Watch for. Passing a person involves the train's length; passing a platform involves both lengths. A moored boat does not have the same relative motion as a drifting one.

Algebra

Identities, factorisation and remainder theorem

01 Recognise square identities

$$a^2 - b^2 = (a - b)(a + b), \quad (a + b)^2 = a^2 + 2ab + b^2$$

The cross term is essential in a square.

02 Factor sums and differences of cubes

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

The signs in the quadratic factor reverse for the middle term.

03 Use the remainder theorem

$$f(x) = (x - a)q(x) + f(a)$$

Remainder on division by x - a is f(a); x - a is a factor exactly when f(a) = 0.

Quick method. Look for a common factor, a difference of squares or a sum/difference of cubes before long division. For divisor ax - b, test x = b/a, with a nonzero.

EXAMPLE 1 **Find the remainder when x^3 - 4x + 7 is divided by x - 2.**

Evaluate the polynomial at 2 rather than dividing it.

$$f(2) = 8 - 8 + 7 = 7$$

EXAMPLE 2 **Factor x^3 - 8.**

Recognise a difference of cubes with b = 2.

$$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

Watch for. A remainder for a linear divisor is constant. A repeated root needs more than the single condition f(a) = 0.

Algebra

Polynomial HCF, LCM and rational expressions

01 Choose minimum and maximum factor powers

HCF : minimum factor powers

LCM : maximum factor powers

Use irreducible/common factors consistently; polynomial HCF is commonly taken monic.

02 Cancel factors while retaining exclusions

$$\frac{(x-a)g(x)}{(x-a)h(x)} = \frac{g(x)}{h(x)}$$

Cancellation is valid only where the original denominator is nonzero.

03 Use the monic HCF-LCM product relation

$$\gcd(f, g)\,\mathrm{lcm}(f, g) = fg$$

For monic polynomials under the usual monic HCF/LCM convention.

Quick method. Factor each polynomial completely first. Cancel factors, not separate terms. Carry excluded values from the original rational expression to the final answer.

EXAMPLE 1 **Find the monic HCF and LCM of $x^2 - 1$ and $x^2 - 2x + 1$.**

Factor as (x - 1)(x + 1) and (x - 1) squared; take minimum and maximum powers.

$$H = x - 1, \quad L = (x-1)^2(x+1)$$

EXAMPLE 2 **Simplify $(x^2 - 9)/(x^2 - 3x)$.**

Factor numerator and denominator, cancel x - 3, and preserve both original exclusions.

$$\frac{x+3}{x}, \quad x \neq 0, 3$$

Watch for. Simplification does not restore a removed denominator zero. For general non-monic polynomials the product identity can differ by a nonzero constant factor.

Algebra

Quadratic roots and parameter conditions

01 Solve a quadratic using its discriminant

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a not zero; real roots require discriminant D >= 0.

02 Use root sums and products

$$\alpha + \beta = -b/a, \quad \alpha\beta = c/a$$

For real-coefficient quadratics; this edition focuses on real roots as specified in the syllabus.

03 Form a symmetric expression in roots

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Use symmetric root expressions before solving the roots individually.

Quick method. A given root determines a parameter by substitution. Equal real roots require D = 0. For two positive real roots, combine D >= 0 with positive sum and product.

EXAMPLE 1 **If the roots of x^2 - 5x + 6 are alpha and beta, find alpha squared + beta squared.**

The sum is 5 and product 6. Use the identity directly.

$$\alpha^2 + \beta^2 = 25 - 12 = 13$$

EXAMPLE 2 **For which real k does x^2 - 6x + k have equal roots ?**

Set its discriminant to zero; the repeated root is 3.

$$36 - 4k = 0 \Rightarrow k = 9$$

Watch for. A zero discriminant means a repeated root, not no root. If the leading coefficient can vanish, check that separate linear case.

Algebra

Simultaneous equations and word problems

01 Translate a pair of linear equations

$$a_1x + b_1y = c_1, \quad a_2x + b_2y = c_2$$

A nonzero coefficient determinant gives one intersection and a unique solution.

02 Check the determinant for a unique solution

$$a_1b_2 - a_2b_1 \neq 0$$

If the determinant is zero, inspect the constant ratios for coincidence or inconsistency.

03 Keep the age difference constant

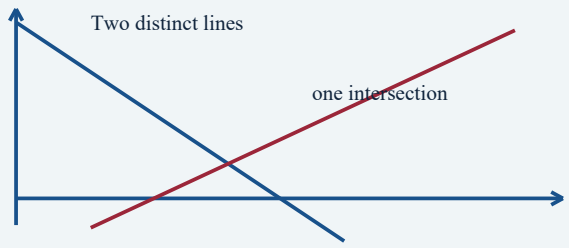
$$\text{future age} = \text{present age} + t$$

Two people's age difference remains constant as the same time passes.

Quick method. Define the unknown quantities and units first. Add/subtract equations to eliminate a variable; reject negative lengths, impossible ages or noninteger counts when the context requires it.

EXAMPLE 1 Adult tickets cost 50 and child tickets 30. Twelve tickets earn 480. Find the numbers of each.

Let adults be a and children 12 - a. Then 50a + 30(12 - a) = 480.



A unique solution is a line intersection.

Parallel distinct lines give no solution.

Coincident lines give infinitely many.

$$a = 6, \quad c = 6$$

EXAMPLE 2 A father is three times his son's age. After 10 years he will be twice as old. Find their present ages.

Write F = 3S and F + 10 = 2(S + 10). Eliminate F.

$$S = 10, \quad F = 30$$

Watch for. Translate more than and less than carefully. Algebraic roots are only candidates until checked against the word problem.

Algebra

Sets, conditional identities and graphical inequalities

01 Count a union without double counting

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Both sets lie in the same universal set; neither = total - union.

02 Use a conditional reciprocal identity

$$x + 1/x = a \Rightarrow x^2 + 1/x^2 = a^2 - 2$$

x is nonzero; use the stated condition before expanding a larger expression.

03 Intersect graphical half-planes

$$ax + by \leq c$$

Draw its boundary, test a point and intersect the required half-planes.

Quick method. For a bounded feasible polygon and a linear objective, compare vertices. For sets, draw the overlap before putting totals into regions.

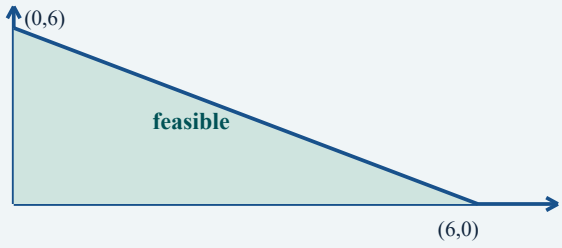
EXAMPLE 1 **In a group of 50, 30 like tea, 25 coffee and 10 both. How many like neither ?**

The union contains 30 + 25 - 10 = 45 people. Subtract it from 50.

$$50 - 45 = 5$$

EXAMPLE 2 **Maximise x + 2y with x >= 0, y >= 0 and x + y <= 6.**

Test the three vertices (0,0), (6,0) and (0,6).



Intersect all required half-planes.

For this bounded triangle, test all vertices.

The objective x + 2y peaks at (0,6).

$$\max(0, 6, 12) = 12 \quad \text{at } (0, 6)$$

Watch for. Multiplying an inequality by a negative reverses its sign. A strict boundary is excluded; a removed denominator zero remains excluded.

Trigonometry

Ratios, exact values and complementary angles

01 Define ratios in a right triangle

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Use a right triangle for acute theta; tangent requires nonzero cosine.

02 Use complementary angles

$$\sin(90^\circ - \theta) = \cos\theta, \quad \tan(90^\circ - \theta) = \cot\theta$$

Complementary-angle identities apply where the functions are defined.

03 Recall exact values at standard angles

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 45^\circ = 1$$

The quick-reference sheet gives the full standard-angle table.

Quick method. For an acute angle and a given ratio, draw a right triangle with convenient side lengths. Use Pythagoras to find the remaining side.

EXAMPLE 1 **If tan theta = 3/4 for acute theta, find sin theta and cos theta.**

Choose opposite 3 and adjacent 4; the hypotenuse is 5.

$$\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$$

EXAMPLE 2 **Evaluate sin squared 30 degrees + cos squared 60 degrees + tan squared 45 degrees.**

Substitute the exact values 1/2, 1/2 and 1 before squaring.

$$\frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$$

Watch for. Tangent at 90 degrees and cotangent at 0 degrees are undefined; they are not zero. Exact surds are usually more useful than rounded decimals.

Trigonometry

Identities and efficient simplification

01 Use the Pythagorean identities

$$\sin^2\theta+\cos^2\theta=1,\quad \sec^2\theta-\tan^2\theta=1$$

Also cosec squared theta - $\cot^2$ theta = 1, where defined.

02 Pair complementary denominators

$$\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\sec^2\theta$$

The denominators must be nonzero.

03 Square a sine-cosine sum

$$(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$$

For acute theta both sine and cosine are positive.

Quick method. Convert mixed secant/cosecant expressions into sine and cosine. Rationalise paired denominators and use the Pythagorean identity to simplify.

EXAMPLE 1 **If sec theta + tan theta = 3, find sec theta - tan theta.**

Their product is sec squared minus tan squared = 1.

$$\sec\theta-\tan\theta=\frac{1}{3}$$

EXAMPLE 2 **If sin theta + cos theta = the square root of (3/2), find sin theta cos theta.**

Square the given relation and subtract 1.

$$2\sin\theta\cos\theta=\frac{1}{2}\Rightarrow\sin\theta\cos\theta=\frac{1}{4}$$

Watch for. Cancel common factors only. Cancelling a term from a sum is invalid, and dividing by a trig function can exclude zero cases.

Trigonometry

Heights, distances and two observation points

01 Relate height to horizontal distance

$$h = d\tan\theta$$

Height h above eye level, horizontal separation d and elevation theta.

02 Use a same-side two-point setup

$$h = \frac{d\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

Same-side ground-level points separated by d, nearer elevation beta > farther elevation alpha.

03 Equate expressions for the same height

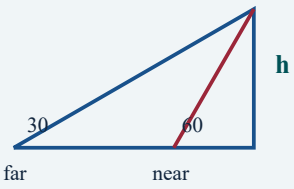
$$h = x\tan\beta = (x+d)\tan\alpha$$

Two points on the same side of a tower, separated by d; nearer angle beta > farther angle alpha.

Quick method. Set the unknown nearer distance to x and use x + d for the farther point. Add observer height only after calculating the height above the observer's horizontal level.

EXAMPLE 1 **Two ground-level points on the same side of a tower are 20 m apart. Elevations are 30 degrees at the farther point and 60 degrees at the nearer. Find the height.**

If the nearer distance is x, then x times square root of 3 = (x + 20)/square root of 3, giving x = 10.



angles in degrees

Same side: far distance = near distance + d.

Write both expressions for the same height.

Opposite-side points need a different setup.

$$h = 10\sqrt{3}\ \text{m}$$

EXAMPLE 2 **An observer whose eye is 1.5 m above ground stands 30 m from a building. The top's elevation is 45 degrees. Find the building height.**

The height above eye level is 30 tan 45 degrees = 30 m. Add 1.5 m.

$$H = 31.5\ \text{m}$$

Watch for. Points on opposite sides require a sum of distances, not the same-side setup. An angle of depression uses a parallel horizontal line.

Geometry

Lines, parallel angles and polygons

01 Use straight-line and full-turn angle sums

$$\text{straight angle} = 180^\circ, \quad \text{angles around a point} = 360^\circ$$

Vertically opposite angles are equal.

02 Count polygon interior and exterior angles

$$S_n = (n-2)180^\circ, \quad \text{regular exterior angle} = 360^\circ/n$$

For an n-sided polygon; the regular formula assumes equal exterior angles.

03 Count a polygon's diagonals

$$\text{diagonals} = \frac{n(n-3)}{2}$$

Count each diagonal once; n >= 3.

Quick method. Mark equal corresponding/alternate angles only after establishing parallel lines. A transversal's co-interior angles sum to 180 degrees.

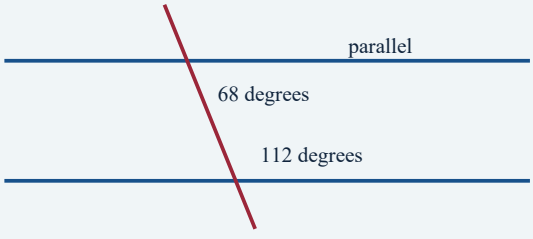
EXAMPLE 1 Each interior angle of a regular polygon is 150 degrees. Find its number of sides.

The exterior angle is 30 degrees; a full turn contains n of them.

$$n = 360/30 = 12$$

EXAMPLE 2 Two parallel lines are cut by a transversal. One co-interior angle is 68 degrees. Find the other.

Same-side interior angles are supplementary.



Co-interior angles add to 180 degrees.

Alternate and corresponding angles are equal.

These rules require parallel lines.

$$180^\circ - 68^\circ = 112^\circ$$

Watch for. The converse angle tests can establish parallelism, but a diagram that merely looks parallel is not evidence. Exterior and interior polygon angles are supplementary.

Geometry

Triangles: congruence, centres and inequalities

01 Apply angle and side inequalities

$$A + B + C = 180^\circ, \quad |a - b| < c < a + b$$

For a nondegenerate triangle; an exterior angle equals the sum of the opposite interior angles.

02 Locate the centroid on a median

$$AG : GM = 2 : 1$$

The centroid G divides every median from vertex A to side midpoint M.

03 Locate the circumcentre of a right triangle

$$R_{\text{right}} = \frac{\text{hypotenuse}}{2}$$

In a right triangle, the circumcentre is the hypotenuse midpoint.

Quick method. Congruence tests are SSS, SAS, ASA/AAS and RHS. The incenter is the intersection of angle bisectors; circumcenter of perpendicular bisectors; orthocenter of altitudes; centroid of medians.

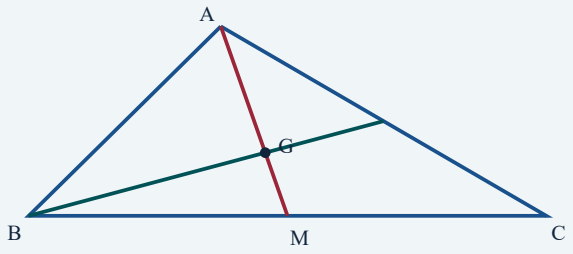
EXAMPLE 1 Two sides of a triangle are 5 and 9. If the third side is an integer, how many values can it have ?

Use the strict inequality $4 < c < 14$, giving integers 5 through 13.

9 possible values

EXAMPLE 2 A median has length 12 cm. How far is the centroid from its vertex ?

The vertex-to-centroid portion is two-thirds of the whole median.



Medians meet at the centroid G.

The vertex-to-centroid portion is two-thirds.

AG:GM = 2:1 on median AM.

$$AG = \tfrac{2}{3}(12) = 8 \text{ cm}$$

Watch for. AAA gives similarity, not congruence. SSA is not a universal congruence test. Triangle inequalities are strict for a genuine triangle.

Geometry

Similarity, proportional segments and area ratios

01 Square the corresponding side ratio

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k \Rightarrow \frac{A_1}{A_2} = k^2$$

Similar triangles have proportional sides and equal corresponding angles.

02 Use a line parallel to a triangle side

$$DE \parallel BC \Rightarrow \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

D lies on AB and E on AC of triangle ABC.

03 Compare areas with one common altitude

$$\frac{[ABD]}{[ADC]} = \frac{BD}{DC}$$

D is on BC; both triangles share the altitude from A.

Quick method. Name corresponding vertices in the same order. For equal altitudes, area ratios equal base ratios; for similar triangles, they equal squared side ratios.

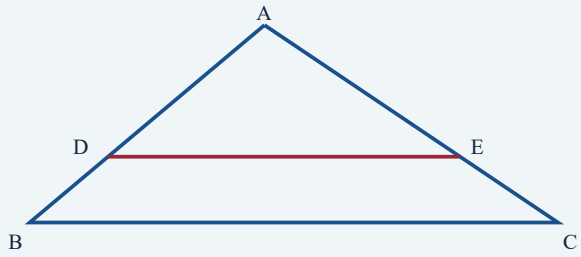
EXAMPLE 1 Similar triangles have corresponding sides in ratio 3:5. Their areas sum to 136. Find the smaller area.

The areas are in ratio 9:25, totalling 34 parts.

$$A_{\text{small}} = 136\frac{9}{34} = 36$$

EXAMPLE 2 In triangle ABC, DE is parallel to BC, AD = 6, DB = 3 and BC = 12. Find DE.

AB = 9 and the similarity scale is AD/AB = 2/3.



DE parallel to BC gives similar triangles.

Side ratio 2:3 becomes area ratio 4:9.

Corresponding vertices must stay in order.

$$DE = 12\frac{6}{9} = 8$$

Watch for. A linear scale factor is not an area factor. A parallel segment inside a triangle creates a smaller similar triangle, not generally one of equal area.

Geometry

Parallelograms, rectangles, rhombi and squares

01 Use a perpendicular height or diagonals

$$A_{\text{parallelogram}} = bh, \quad A_{\text{rhombus}} = \tfrac{1}{2}d_1 d_2$$

Use perpendicular height, or the rhombus's perpendicular diagonals.

02 Link side lengths and diagonals

$$d^2_{\text{rectangle}} = l^2 + b^2, \quad d_{\text{square}} = a\sqrt{2}$$

A rectangle has equal diagonals; a rhombus has perpendicular diagonals.

03 Use adjacent angles in a parallelogram

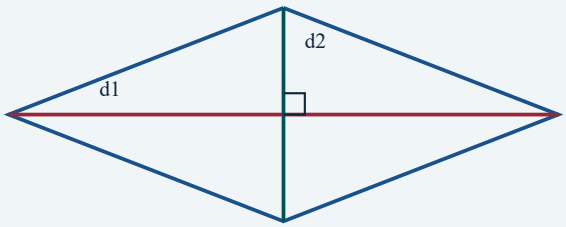
adjacent parallelogram angles sum to 180°

Opposite sides and angles are equal; diagonals bisect each other.

Quick method. A parallelogram with one right angle is a rectangle; with all sides equal is a rhombus; with both properties is a square. Diagonal properties often identify the right classification.

EXAMPLE 1 A rhombus has diagonals 10 and 24 cm. Find its side and area.

Half-diagonals form a right triangle with legs 5 and 12; the side is 13.



Rhombus diagonals are perpendicular.

Each diagonal bisects the other.

Half-diagonals and a side form a right triangle.

$$a = 13 \text{ cm}, \quad A = \tfrac{1}{2}(10)(24) = 120 \text{ cm}^2$$

EXAMPLE 2 A rectangle has perimeter 34 cm and diagonal 13 cm. Find its area.

l + b = 17 and l squared + b squared = 169. Subtract from the square of the sum to get 2lb = 120.

$$A = lb = 60 \text{ cm}^2$$

Watch for. A rhombus need not have equal diagonals, and a rectangle need not have perpendicular diagonals. Every square has both sets of properties.

Geometry

Circles, chords and cyclic angles

01 Relate a central angle to an inscribed angle

$$\angle AOB = 2\angle ACB$$

O is the centre; both angles subtend the same arc AB, with C on the other arc.

02 Use opposite angles in a cyclic quadrilateral

$$\angle A + \angle C = 180^\circ$$

Opposite angles of a cyclic quadrilateral are supplementary.

03 Bisect a chord with a perpendicular radius

$$d^2 + (c/2)^2 = r^2$$

A perpendicular from a circle's centre bisects a chord of length c; d is its distance from the centre.

Quick method. An angle in a semicircle is a right angle. Equal chords subtend equal central angles and lie equally far from the centre; use the converse carefully.

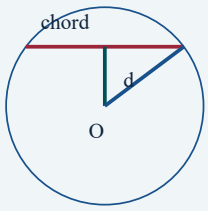
EXAMPLE 1 **A cyclic quadrilateral has one angle 76 degrees. Find its opposite angle.**

Subtract from 180 degrees.

$$C = 104^\circ$$

EXAMPLE 2 **A circle has radius 13 cm and a chord 5 cm from its centre. Find the chord length.**

The half-chord is a right-triangle leg: square root of (169 - 25) = 12.



A centre perpendicular bisects the chord.

Half-chord, distance and radius form a right triangle.

$$c = 2(12) = 24 \text{ cm}$$

Watch for. The central-angle theorem needs the same arc, not just the same endpoints with the wrong arc choice. The centre need not lie on the drawn angle's interior.

Geometry

Tangents, secants and loci

01 Use equal tangents and a contact radius

$$PA = PB, \quad OA \perp PA$$

Tangents from external P have equal lengths; radius OA is normal to the tangent at contact point A.

02 Relate a tangent to a secant

$$PT^2 = PA \cdot PB$$

P is external, PT a tangent and P-A-B a secant in that order.

03 Multiply near and far secant distances

$$PA \cdot PB = PC \cdot PD$$

Two secants from the same external point P; use near times far on each secant.

Quick method. Equidistant from two fixed points gives their perpendicular bisector; from two intersecting lines gives their angle bisectors; at fixed distance from a point gives a circle.

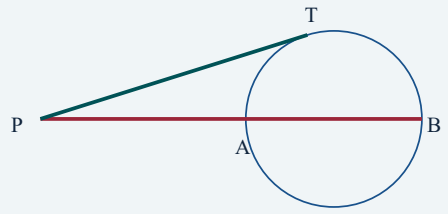
EXAMPLE 1 An external point is 13 cm from the centre of a circle of radius 5 cm. Find a tangent length.

The contact radius and tangent form a right angle.

$$PT = \sqrt{13^2 - 5^2} = 12 \text{ cm}$$

EXAMPLE 2 A secant from P meets a circle at A and B with PA = 4 and AB = 5. Find the tangent length from P.

The full secant length PB is 4 + 5 = 9, not 5.



Use PA times PB: near distance times far.

The tangent length is squared.

Do not substitute the internal segment AB in place of the whole secant PB.

$$PT = \sqrt{4 \cdot 9} = 6$$

Watch for. A line from the centre to a chord is not necessarily a bisector unless it is perpendicular (or passes through the chord midpoint). Loci may need restrictions imposed by the original problem.

Mensuration

Plane areas, perimeters and sectors

01 Use a perpendicular height

$$A_{\text{triangle}} = \tfrac{1}{2}bh, \quad A_{\text{trapezium}} = \tfrac{1}{2}(a+b)h$$

a,b are parallel sides of the trapezium; h is perpendicular separation.

02 Separate circumference, area and sector

$$C = 2\pi r, \quad A_{\text{circle}} = \pi r^2, \quad A_{\text{sector}} = \frac{\theta}{360^\circ}\pi r^2$$

Theta is the central angle in degrees.

03 Recognise special triangle areas

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}, \quad A_{\text{equilateral}} = \frac{\sqrt{3}}{4}a^2$$

Heron's semiperimeter s = (a + b + c)/2; use nondegenerate triangle sides.

Quick method. For a semicircle, add its diameter to the curved arc when finding total perimeter. For a ring or path, subtract the inner area from the outer area.

EXAMPLE 1 **Find the total perimeter of a semicircle of radius 7 cm, using pi = 22/7.**

Add the half-circumference and the diameter.

$$P = \pi r + 2r = 22 + 14 = 36 \text{ cm}$$

EXAMPLE 2 **Find the area of a triangle with sides 13,14,15 cm.**

s = 21; insert into Heron's formula.

$$\Delta = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84 \text{ cm}^2$$

Watch for. Perimeter scales with length; area with length squared. Do not use a slant side as a perpendicular height.

Mensuration

Cubes, cuboids and space diagonals

01 Separate volume from total surface area

$$V = lbh, \quad TSA = 2(lb + bh + hl)$$

A closed rectangular cuboid; open boxes omit the missing faces.

02 Find the internal space diagonal

$$d = \sqrt{l^2 + b^2 + h^2}, \quad V_{\text{cube}} = a^3$$

d is the cuboid's internal space diagonal.

03 Identify the surfaces being counted

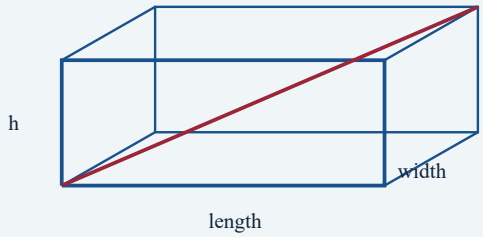
$$TSA_{\text{cube}} = 6a^2, \quad LSA_{\text{cuboid}} = 2h(l + b)$$

Lateral area omits the top and bottom for height h.

Quick method. List which faces exist before calculating surface area. For a recast solid, equate volumes if material is conserved; surface area generally changes.

EXAMPLE 1 **Find the space diagonal of a cuboid of dimensions 3,4,12 cm.**

Apply the three-dimensional Pythagorean formula.



The space diagonal crosses the interior.

Its square is l squared + b squared + h squared.

Open and closed boxes have different areas.

$$d = \sqrt{9 + 16 + 144} = 13 \text{ cm}$$

EXAMPLE 2 **A closed cube has total surface area 216 square cm. Find its volume.**

6a squared = 216 gives a = 6 cm; then cube the edge.

$$V = 6^3 = 216 \text{ cm}^3$$

Watch for. A space diagonal differs from a face diagonal. Capacity in litres is not the same numerical unit as cubic centimetres.

Mensuration

Cylinders, hollow pipes and capacity

01 Use the cylinder's radius and height

$$V = \pi r^2 h, \quad CSA = 2\pi rh, \quad TSA = 2\pi r(h + r)$$

For a right circular cylinder; TSA includes both circular ends.

02 Subtract the cylindrical void

$$V_{\text{material}} = \pi(R^2 - r^2)h$$

Hollow cylinder with outer radius R, inner radius r and R > r.

03 Convert capacity and cubic units

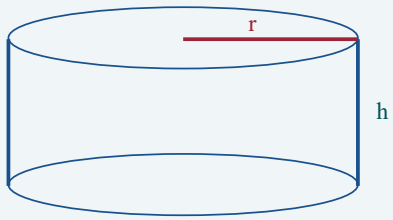
$$1\ \text{L} = 1000\ \text{cm}^3, \quad 1\ \text{m}^3 = 1000\ \text{L}$$

Convert all linear measurements into a common unit before cubing them.

Quick method. An open cylindrical tank's area is the curved side plus only its base. A pipe may require internal, external, end-ring or combined area; identify the requested surface.

EXAMPLE 1 **A cylinder has radius 7 cm and height 10 cm. Find its volume using pi = 22/7.**

Insert radius and height in pi r squared h.



Volume = base area times perpendicular height.

Curved area unwraps into a rectangle.

Closed cylinder: add both circular ends.

$$V = 1540\ \text{cm}^3 = 1.54\ \text{L}$$

EXAMPLE 2 **Find the material volume of a hollow cylinder with outer radius 5, inner radius 3 and length 10 cm.**

Subtract the inner cylindrical void from the outer cylinder.

$$V = \pi(25 - 9)(10) = 160\pi\ \text{cm}^3$$

Watch for. Changing diameter changes radius by the same factor, but volume at fixed height changes by its square.

Mensuration

Cones, slant height and similar sections

01 Distinguish slant and perpendicular heights

$$l = \sqrt{r^2 + h^2}, \quad V = \frac{1}{3}\pi r^2 h$$

For a right circular cone; l is slant height and h is perpendicular height.

02 Identify curved and total cone surfaces

$$CSA = \pi r l, \quad TSA = \pi r(l + r)$$

Total surface area includes the base; an open conical tent usually does not.

03 Cube the linear scale of similar cones

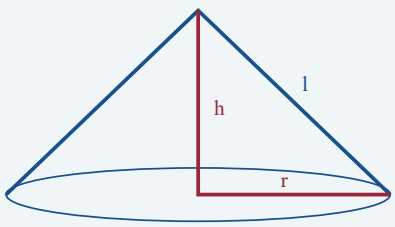
$$\frac{r_1}{r_2} = \frac{h_1}{h_2} = k \Rightarrow \frac{V_1}{V_2} = k^3$$

For similar cones cut by planes parallel to the base.

Quick method. Draw the axial cross-section: its right triangle links r,h,l. A stated full vertex angle is twice the angle between the axis and a generator.

EXAMPLE 1 A cone has radius 3 cm and height 4 cm. Find its curved surface area and volume.

The slant height is 5 by Pythagoras. Use the appropriate length in each formula.



h, r and l form a right triangle.

Slant height belongs in curved surface area.

Perpendicular height belongs in volume.

$$CSA = 15\pi \text{ cm}^2, \quad V = 12\pi \text{ cm}^3$$

EXAMPLE 2 A small similar cone cut from the top has half the original cone's height. What fraction of the original volume remains ?

The small cone's linear scale is 1/2, so its volume fraction is 1/8. Subtract from 1.

$$\text{remaining fraction} = \frac{7}{8}$$

Watch for. Do not use slant height in the volume formula. Equal base and equal volume do not mean a cone and cylinder have equal height.

Mensuration

Spheres, hemispheres and recasting

01 Use sphere volume and surface area

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3, \quad A_{\text{sphere}} = 4\pi r^2$$

A sphere has no flat base.

02 Include the hemisphere's flat base

$$V_{\text{hemisphere}} = \frac{2}{3}\pi r^3, \quad CSA = 2\pi r^2, \quad TSA = 3\pi r^2$$

The hemisphere's total surface area includes its circular base.

03 Conserve volume when recasting spheres

$$r_1^3 + r_2^3 + \cdots = R^3$$

For spheres melted into a single sphere with no loss, cancel the common factor 4pi/3.

Quick method. Use volume conservation for melting/recasting. If radius scales by k, area scales by k squared and volume by k cubed.

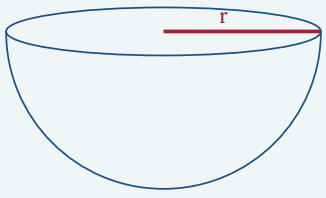
EXAMPLE 1 **Eight spheres of radius 1 cm are melted into one sphere. Find its radius.**

Equate total material volume and cancel 4pi/3.

$$R^3 = 8 \Rightarrow R = 2 \text{ cm}$$

EXAMPLE 2 **A cone and a hemisphere have equal bases and equal volumes. Find cone height in terms of their shared radius r.**

Equate one-third pi r squared h with two-thirds pi r cubed.



A hemisphere has one curved surface

and one flat circular base.

Add the base only for total surface area.

$$h = 2r$$

Watch for. A shared face at a join between solids is internal and must not be counted as exterior surface. A hemisphere's curved and total areas differ.

Mensuration

Composite figures, field-book areas and exposed surfaces

01 Add and subtract component areas

$$A_{\text{composite}} = \sum A_{\text{pieces}} - \sum A_{\text{removed}}$$

Choose nonoverlapping pieces or explicitly correct overlaps.

02 Use parallel offsets in a field-book strip

$$A_{\text{strip}} = \tfrac{1}{2}(y_1 + y_2)d$$

Trapezoidal field-book strip: parallel offsets y1,y2, baseline separation d.

03 Convert land and area units

$$1\ \text{ha} = 10000\ \text{m}^2, \quad 1\ \text{m}^2 = 10000\ \text{cm}^2$$

Square a length conversion for area; cube it for volume.

Quick method. Draw and label the component shapes before calculating. For combined solids, use volume addition but count only the surfaces actually exposed.

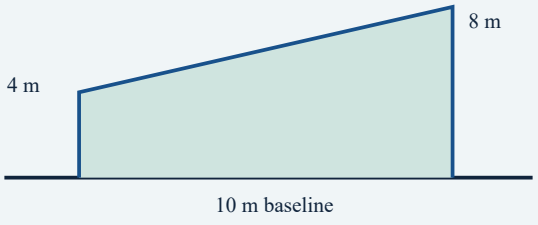
EXAMPLE 1 A 20 m by 14 m rectangle has a semicircle attached externally to a 14 m side. Find the combined area using pi = 22/7.

Add the rectangle area and the semicircle area of radius 7 m; the join has no overlap area.

$$A = 280 + 77 = 357\ \text{m}^2$$

EXAMPLE 2 Offsets 4 m and 8 m lie on the same side of a straight baseline, 10 m apart. Find the trapezoidal strip area.

Average the two offsets and multiply by the baseline separation.



Parallel offsets form a trapezoidal strip.

Area = average offset times baseline length.

Subdivide longer field-book figures into strips.

$$A = \frac{4+8}{2}(10) = 60\ \text{m}^2$$

Watch for. A wire bent into a new shape conserves total length, not area. A field-book area may require strips on both sides of the baseline.

Statistics

Frequencies, histograms, polygons and pie charts

01 Distinguish simple and cumulative frequencies

$$N = \sum_i f_i, \quad F_j = \sum_{i \le j} f_i$$

N is total frequency; Fj is the cumulative frequency up to class j.

02 Use density for unequal class widths

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Use density heights for unequal-width histogram classes so bar areas represent frequencies.

03 Convert a frequency to a sector angle

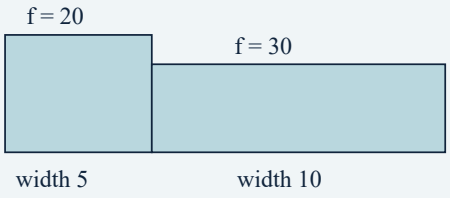
$$\theta_i = 360^\circ \frac{f_i}{N}$$

A pie-chart angle is proportional to that category's frequency.

Quick method. A histogram uses adjacent numerical class intervals. A bar chart compares discrete categories, usually with gaps. A frequency polygon joins class-midpoint frequency points; an ogive uses cumulative frequencies.

EXAMPLE 1 A class of width 5 has frequency 20; another of width 10 has frequency 30. Compare their density heights.

Divide each frequency by its own width.



Unequal class widths require density heights.

First height: 20/5 = 4; second: 30/10 = 3.

Bar areas remain proportional to frequencies.

$$h_1 = 4, \quad h_2 = 3 \Rightarrow h_1 : h_2 = 4 : 3$$

EXAMPLE 2 A pie-chart sector measures 72 degrees and represents 30 people. Find the total number represented.

The sector is 72/360 = 1/5 of the full population.

$$N = 30\frac{360}{72} = 150$$

Watch for. Bar height alone is not frequency in an unequal-width density histogram. Class boundaries and class midpoints have different roles.

Statistics

Mean, median, mode and corrections

01 Calculate a mean from values or frequencies

$$\bar{x} = \frac{\sum x_i}{n} = \frac{\sum f_i x_i}{\sum f_i}$$

Use raw data for the first formula and frequency data for the second.

02 Combine groups of unequal sizes

$$\bar{x}_{\text{combined}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Weight group means by group size; a simple average requires equal group sizes.

03 Relate deviations to the mean

$$\sum_{i=1}^{n}(x_i - a) = n(\bar{x} - a)$$

Deviation sums can recover the mean without listing the data.

Quick method. Sort observations before taking the median; for an even count average the two middle values. Mode is the most frequent value and need not be unique. Correct a mean by correcting its total sum first.

EXAMPLE 1 **The mean of 10 numbers is 18. A value 25 was mistakenly recorded as 15. Find the corrected mean.**

The recorded total is 180. Replace the wrong entry by adding 25 - 15 = 10.

$$\bar{x}_{\text{corrected}} = 190/10 = 19$$

EXAMPLE 2 **For n observations, the sum of deviations from 10 is -20, and from 4 is 40. Find n and the mean.**

Subtract the two deviation equations: 6n = 60. Then n(mean - 10) = -20.

$$n = 10, \quad \bar{x} = 8$$

Watch for. A mean need not coincide with an observation, but cannot lie outside the minimum-to-maximum interval. Do not use an unweighted mean of unequal-sized groups.

Arithmetic toolkit and percentage anchors

Recognise the base, the unit and the invariant before selecting a shortcut.

Useful percentage fractions

$$50\% = \tfrac{1}{2}, \quad 25\% = \tfrac{1}{4}, \quad 20\% = \tfrac{1}{5}, \quad 12.5\% = \tfrac{1}{8}$$

More anchors

$$16\tfrac{2}{3}\% = \tfrac{1}{6}, \quad 33\tfrac{1}{3}\% = \tfrac{1}{3}, \quad 66\tfrac{2}{3}\% = \tfrac{2}{3}$$

Successive signed changes

$$\Delta\% = p + q + pq/100$$

Equal up/down percentages

$$\text{net loss}\% = p^2/100$$

Equal-SP profit and loss

$$\text{overall loss}\% = p^2/100 \quad (0 < p < 100)$$

Two-year CI-SI

$$CI - SI = P(r/100)^2$$

Work together

$$T = \frac{ab}{a+b}, \quad \text{net rate} = \sum \text{signed rates}$$

Equal-distance average speed

$$v = \frac{2uv}{u+v} \quad (u, v > 0)$$

Still water / stream

$$u = \frac{v_d + v_u}{2}, \quad v = \frac{v_d - v_u}{2}$$

Mixture mean

$$c = \frac{q_1 c_1 + q_2 c_2}{q_1 + q_2}$$

Repeated replacement

$$C_n = C_0(1 - v/V)^n$$

Capital-time shares

$$S_1 : S_2 = P_1 t_1 : P_2 t_2$$

The equal-SP loss rule needs two items with equal selling prices and equal gain/loss percentages p. The CI-SI rule assumes annual compounding for exactly two years at one fixed rate. Replacement assumes a well-mixed, fixed-volume container.

Algebra and exact trigonometric values

All identities are used only where their expressions are defined. CDS calculations favour exact values.

Standard sines

$$\sin(0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ) = (0, 1/2, 1/\sqrt{2}, \sqrt{3}/2, 1)$$

Standard cosines

$$\cos(0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ) = (1, \sqrt{3}/2, 1/\sqrt{2}, 1/2, 0)$$

Standard tangents

$$\tan(0^\circ, 30^\circ, 45^\circ, 60^\circ) = (0, 1/\sqrt{3}, 1, \sqrt{3})$$

Reciprocal ratios

$$\sec\theta = 1/\cos\theta, \quad \csc\theta = 1/\sin\theta, \quad \cot\theta = 1/\tan\theta$$

Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1, \quad \sec^2\theta = 1 + \tan^2\theta$$

Companion identity

$$\csc^2\theta = 1 + \cot^2\theta$$

Two-point tower setup

$$h = x\tan\beta = (x + d)\tan\alpha$$

Square and cube identities

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Conditional reciprocal sum

$$x + 1/x = a \Rightarrow x^3 + 1/x^3 = a^3 - 3a$$

Quadratic roots

$$\alpha + \beta = -b/a, \quad \alpha\beta = c/a, \quad D = b^2 - 4ac$$

Real-root cases

$$D > 0: \text{ two distinct}; \quad D = 0: \text{ repeated}; \quad D < 0: \text{ none}$$

Linear inequalities

$$ax + by \le c: \quad \text{boundary } ax + by = c$$

The reciprocal definition cot = 1/tan is used only where both sides exist; cot 90 degrees is zero even though tan 90 degrees is undefined. For tower setup, the two points are on the same side at ground level, beta is the nearer angle and d their separation.

Geometry and mensuration at a glance

Use perpendicular heights, correct radii and the exact exposed surfaces. Area and volume have different units.

Triangle area

$$\Delta = \tfrac{1}{2}bh = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = (a+b+c)/2$$

Equilateral triangle

$$h = \frac{\sqrt{3}}{2}a, \quad \Delta = \frac{\sqrt{3}}{4}a^2$$

Trapezium / rhombus

$$A = \tfrac{1}{2}(a+b)h, \quad A_{\text{rhombus}} = \tfrac{1}{2}d_1 d_2$$

Circle / sector

$$A = \pi r^2, \quad C = 2\pi r, \quad s_{\text{arc}} = \frac{\theta}{360^\circ}2\pi r$$

Cyclic quadrilateral

$$A + C = 180^\circ, \quad B + D = 180^\circ$$

Tangent-secant relation

$$PT^2 = PA \cdot PB \quad (P - A - B \text{ on one secant})$$

Cuboid

$$V = lbh, \quad TSA = 2(lb + bh + hl), \quad d = \sqrt{l^2 + b^2 + h^2}$$

Cylinder

$$V = \pi r^2 h, \quad CSA = 2\pi rh, \quad TSA = 2\pi r(h + r)$$

Cone

$$l^2 = r^2 + h^2, \quad V = \tfrac{1}{3}\pi r^2 h, \quad CSA = \pi rl$$

Sphere

$$V = \tfrac{4}{3}\pi r^3, \quad A = 4\pi r^2$$

Hemisphere

$$V = \tfrac{2}{3}\pi r^3, \quad CSA = 2\pi r^2, \quad TSA = 3\pi r^2$$

Similar figures

$$\text{lengths } k: \quad \text{areas } k^2, \quad \text{volumes } k^3$$

In a cone or cylinder, total surface area includes the base(s). Joined faces of a composite solid are internal, so exclude them from exposed area. Similar-figure scale rules require genuinely similar shapes.

Number tests, data and final checks

A last-revision checklist for avoidable arithmetic and interpretation errors.

Divisibility: 2, 5, 10

2 : last digit even; 5 : last digit 0 or 5; 10 : last digit 0

Divisibility: 3 and 9

3 or 9 : digit sum divisible by 3 or 9

Divisibility: 4 and 8

4 : final two digits; 8 : final three digits

Divisibility: 11

alternating digit sum is a multiple of 11, including 0

Common right-triangle triples

$(3, 4, 5)$, $(5, 12, 13)$, $(8, 15, 17)$, $(7, 24, 25)$

Length and capacity

$1\ \text{km} = 1000\ \text{m}, \quad 1\ \text{L} = 1000\ \text{cm}^3$

Area and volume conversion

$1\ \text{m}^2 = 10^4\ \text{cm}^2, \quad 1\ \text{m}^3 = 10^6\ \text{cm}^3$

Weighted mean

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Corrected mean

$$\bar{x}_{\text{new}} = \bar{x}_{\text{old}} + \frac{\text{correct value} - \text{wrong value}}{n}$$

Grouped median estimate

$$\text{Median} = L + \frac{N/2 - C_f}{f}h$$

Grouped mode estimate

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}h$$

Grouped formulas are estimates for suitable continuous class data. The mode formula uses equal class widths and a nonzero denominator: L is the target class lower boundary; h its width; Cf the preceding cumulative frequency; f the median-class frequency; f1 the modal-class frequency and f0,f2 its neighbours. Before choosing an answer, recheck units, excluded values, the percentage base, diagram conditions and whether the result fits the original question.

Statement checks and data sufficiency

Test each statement alone, then combine them if needed. Sufficient means the requested answer is uniquely determined, including a definite yes/no answer when that is the question.

Two roots are not one value

$x^2 = 9 \Rightarrow x = \pm 3; \quad x > 0 \Rightarrow x = 3$

One sum leaves freedom

$x + y = 12: \quad (4, 8), (5, 7), (6, 6)$ all work

Independent second relation

$x + y = 12, \quad x - y = 4 \Rightarrow (x, y) = (8, 4)$

Redundant second relation

$x + y = 12, \quad 2x + 2y = 24$ still do not fix x

Inconsistent relations

$x + y = 12, \quad 2x + 2y = 25$ cannot both hold

Counterexample to a universal claim

$x = \frac{1}{2}: \quad x^2 = \frac{1}{4} < x$

Do not lose sign cases

$\sqrt{x^2} = |x|,$ not always x

Geometry supplies restrictions

$a^2 = 36, \quad a$ is a side length $\Rightarrow a = 6$

A conditional identity needs its domain

$x + 1/x = a$ requires $x \neq 0$

Do not assume that two statements are independent simply because there are two of them. Keep positivity, integrality, order and geometric restrictions from the question. A single valid counterexample disproves a universal assertion; a few successful examples do not prove it. Read the sufficiency answer choices each time: their ordering can change.